



THE OHIO STATE UNIVERSITY

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Analysis of Top Quarks Using a Kinematic Likelihood Method

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Abstract

The ATLAS detector at the LHC is able to measure the trajectory of particles with great precision, however there is some systematic error in measuring the energy. This systematic error can be quantitatively described by what are known as transfer functions. For this work, I have computed a new set of transfer functions, compatible with detector upgrades and a center of mass energy of 13 TeV. These transfer functions were then applied to an analysis of Monte Carlo data to verify their effectiveness.

1 The Standard Model

The Standard Model of particle physics is a quantum field theory that attempts to describe all elementary particles and their interactions. In the Standard Model, there are two types of particles, fermions which are characterized by spin angular momentum $1/2$ and bosons which are characterized by integer spin angular momentum. Spin statistics prohibit two identical fermions from occupying the same state at the same time. No such prohibition exists for bosons. The fermions are then split into two groups; leptons and quarks.

The quarks in the Standard Model come in three generations of doublets. Each doublet has a quark with electric charge equal to $\frac{2}{3}e$ and a quark with charge equal to $-\frac{1}{3}e$. The three doublets consist of the up and down quarks, the charm and strange quark, and the top and bottom quarks. Quarks interact through the strong, weak, and electromagnetic forces. Due to their interaction with the strong force, quarks only exist as bound components of composite particles, known as hadrons, and never as individual particles.

The leptons in the Standard Model also come in three generations of doublets. Each doublet contains a particle with electric charge equal to the charge of the electron and a neutrino which has no electric charge and is almost massless. The three doublets are generally referred to by the name of the charged particle in the doublets which are electrons, muons, and taus respectively. Unlike the quarks, the leptons only interact through the weak and electromagnetic forces. Note that due to having no charge, the neutrinos do not interact through the electromagnetic force.

In the Standard Model the forces between particles are represented or visualized as the exchange of a spin 1 gauge boson. The strength of a particle's interaction with a force is determined by a property called charge. In the electromagnetic interaction the gauge boson, the photon, couples to electric charge. This interaction is illustrated in figure 2. The strong force is mediated by 8 gluons which couple to color charge. The W^\pm and Z bosons mediate the weak force which couples to weak charge. The weak force is unique in that it is a chiral force, which means that it only interacts with left-handed particles and that it allows particles to change their flavor [1].

In the simplest version of the Standard Model, all particles are massless. However, this is not an accurate description of reality, so the Standard Model introduces the Higgs boson

Three Generations of Matter (Fermions)									
	I	II	III						
mass→	8 MeV	1.24 GeV	172.5 GeV	0	126.7 GeV				
charge→	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0				
spin→	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0				
name→	u	c	t	γ	H				
	up	charm	top	photon	Higgs				
Quarks	6 MeV	95 MeV	4.2 GeV	0	0				
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	0				
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	2				
	d	s	b	g	G				
	down	strange	bottom	gluon	Graviton				
Leptons	<2 eV	<0.19 MeV	<18.2 MeV	0	0				
	0	0	0	0	0				
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1				
	ν_e	ν_μ	ν_τ	Z	W				
	electron neutrino	muon neutrino	tau neutrino	weak force	weak force				
	0.511 MeV	106 MeV	1.78 GeV	80.4 GeV					
	-1	-1	-1	1	1				
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1				
	e	μ	τ	W					
	electron	muon	tau	weak force					

Figure 1:
The Standard Model
Particles ¹

¹Photo Credit: InspireHEP

to give particles their mass. To be capable of making predictions at all energy scales, the Standard Model must satisfy a condition known as renormalizability. It can be shown that any theory with interacting spin 1 particles is renormalizable as long as it satisfies a gauge symmetry. The electromagnetic force has a $U(1)$ symmetry and the strong force has a $SU(3)$ symmetry which guarantee their renormalizability. However, the $SU(2)$ symmetry of the weak force is broken by the masses of the W and Z bosons. The Higgs mechanism is a workaround that allows for the W and Z bosons to be massive and still respect a $SU(2)$ symmetry. The Higgs mechanism introduces a complex scalar particle called the Higgs boson. The Higgs boson interacts with all of the massive particles in the Standard Model and has a self interaction that results in a non-zero expectation value for the Higgs field in the vacuum of the theory. This non-zero vacuum expectation value allows for the interactions with the Higgs boson to give masses to all particles in the Standard Model while also respecting all of the gauge symmetries required for renormalizability [2].

As a quantum theory, the Standard Model makes predictions of the lifetime of unstable particles and cross sections which describe how likely it is for particles to scatter into another state. Due to the theoretical difficulties involved with interacting field theories, these quantities cannot be computed exactly and perturbation theory must be used. To simplify these calculations, Feynman introduced diagrams to represent each term in the perturbation series. Feynman diagrams allow us to visualize interactions as particles emitting and absorbing virtual particles which then decay into the final state. For example, figure 2 displays a Feynman diagram which describes an electron positron pair annihilating each other to form a photon which decays to a muon anti-muon pair.

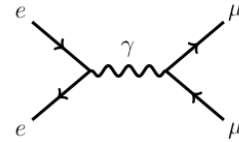


Figure 2:
Feynman Diagram for $e^+e^- \rightarrow \mu^+\mu^-$

The predictions of the Standard Model can be tested at particle accelerators such as the Large Hadron Collider (LHC) in Switzerland where protons are collided into each other at nearly the speed of light. Protons are composed of two up quarks, a down quark, and a sea of virtual particles that binds them together. When the protons collide, their component particles can interact which produces other particles that we are interested in studying. These particles can also decay after being produced. If quarks are produced in an event, the strong force will cause them to bind into composite particles called jets. The resulting jets and leptons in each event are measured in the detectors surrounding the point of collision in the accelerator.

For this study, we will be focused on events where pairs of top quarks are produced. The top quark is the heaviest of the quarks and this puts it in a rather unique position. Due to the strong interaction, quarks only exist as parts of composite particles known as hadrons and never exist all on their own. However, due to its large mass, the top quark is so

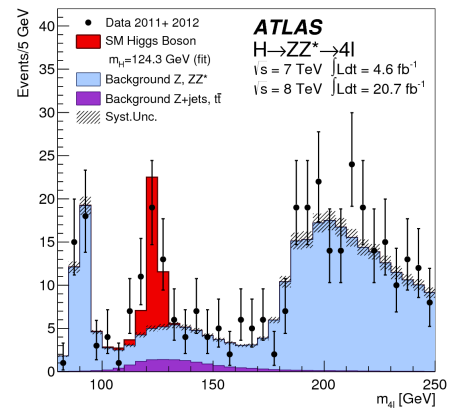


Figure 3: Higgs Discovery Mass Plot

²Photo Credit: quantumdiaries.org

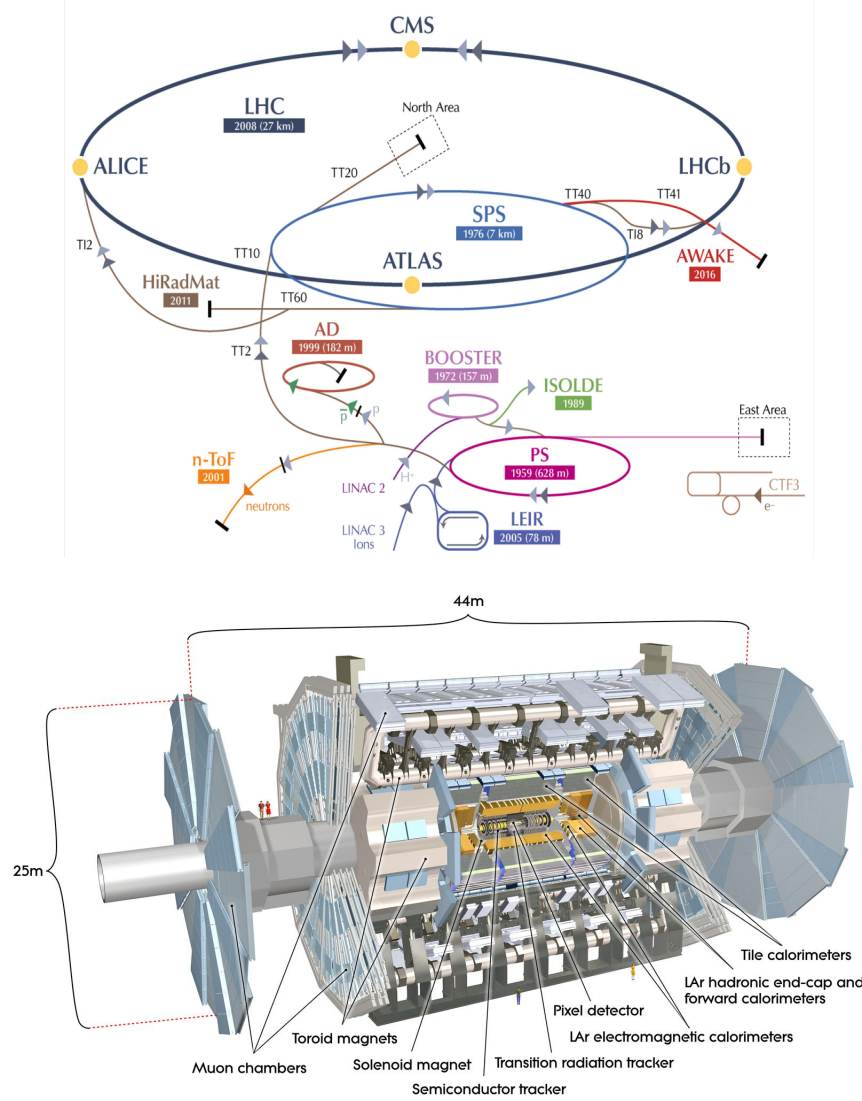


Figure 4: The LHC and the ATLAS Detector ³

unstable that it decays before binding with other quarks to form a hadron. So the top quark presents a unique opportunity to study a quark outside of a hadron. Understanding top quark production is also very important in studying the Higgs boson. The discovery of the Higgs boson in 2012 was through the decay of the Higgs into other bosons, so the interaction of the Higgs with quarks and leptons has not yet been measured [3]. One possible decay channel to study these interactions is the decay of a pair of Higgs bosons to a pair of b quarks and a pair of W bosons. However, this results in the same final state particles as the decay of a top anti-top pair. So $t\bar{t}$ events are a sizeable background when one wishes to study diHiggs events. To perform this kind of study, it would be necessary to ensure that we have a thorough understanding of the top quark's interactions so this background can be eliminated.

³Photo Credit: InspireHEP.net

2 The ATLAS Detector

The ATLAS detector is one of five detectors at the LHC. The detector consists of an inner-detector tracking system surrounded by a thin superconducting solenoid, electromagnetic and hadronic calorimeters, and a muon spectrometer which incorporates three large superconducting toroid magnets. Figure 4 displays the schematics of the ATLAS detector and its position inside the Large Hadron Collider.

The electromagnetic calorimeter contains liquid argon which absorbs energy from bremsstrahlung as charged particles pass through it. Combined with information about their path from the transition radiation tracker, it is capable of measuring the energy of electrons. The jets and muons are not detected in the electromagnetic calorimeter since the intensity of bremsstrahlung radiation emitted is proportional to $\frac{1}{mass^5}$ and muons and jets are hundreds of times heavier than electrons.

The hadronic calorimeter consists of scintillator tiles surrounding the outside of the electromagnetic calorimeter. The electromagnetic calorimeter is large enough that no electrons make it through, so the only charged particles that make it to the hadronic calorimeter are jets and muons. As the jets collide with the layers of the calorimeter, they will decay into a shower of other particles which can be measured with the detector. These hadron showers are illustrated in figure 5. The muons do not emit these hadronic showers which allows the detector to distinguish between the muons and jets.

The outermost part of the ATLAS detector consists of the muon spectrometer. The muon spectrometer consists of high precision tracking chambers immersed in a magnetic field generated by superconducting toroids. Due to the magnetic field, the muons' path will curve as they pass through the tracking chambers. By measuring the curvature of this path, it is possible to determine the energy of the muon that passed through the chamber.

The ATLAS detector is not capable of measuring neutrinos, so their energy must be reconstructed indirectly. The LHC collides protons directly into each other, so conservation of momentum requires that the net momentum transverse to the beamline sums to zero. Using this requirement, we can associate the missing transverse energy in each event with a neutrino.

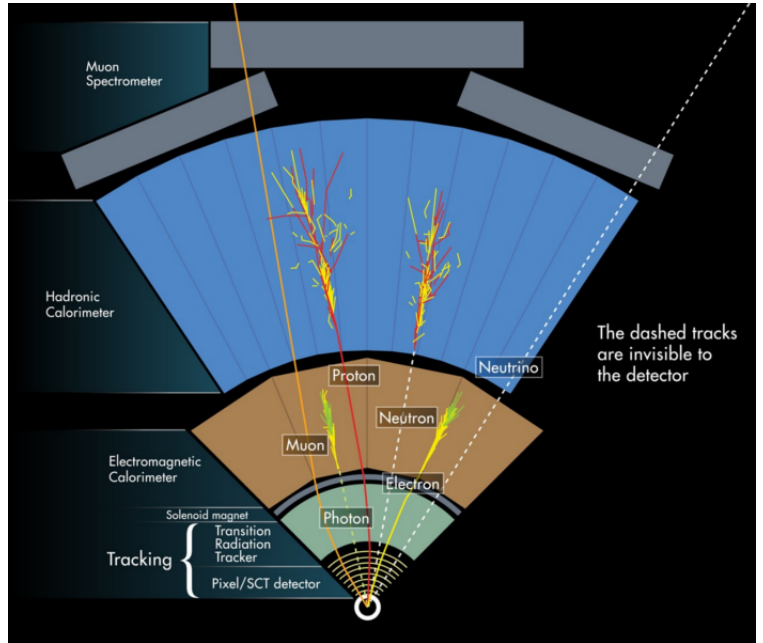


Figure 5: ATLAS detector with particle tracks ⁴

⁴Photo Credit: The ATLAS Collaboration

3 Kinematic Likelihood Method

In any analysis, the problem of how our detector variables relate to the underlying theory must be solved. However, the Standard Model, and extensions that we may want to test, are formulated in terms of quarks and not jets which we actually measure. The ATLAS detector directly measures electrons and muons so those particles are easier to match to the theory than jets. Neutrinos also must be reconstructed from the missing transverse energy in some way.

Previously, a kinematic likelihood method was developed to solve this problem for the case of a pair of b quarks and a pair of W bosons being produced by the decay of a top quark pair. The W bosons then can decay to a pair of light quarks or a lepton neutrino pair [4]. Kinematic fitting works by assuming the measured particles were produced by a $t\bar{t}$ event and then maximizes the probability of observing our measurement as a function of the energies and transverse momentum of the truth level objects. In addition to improving energy measurements, the fitting allows us to match jets to the quarks that produced them which is generally a difficult problem to solve. Also when applied to data, the fit gives the probability that the event was produced by a $t\bar{t}$ event which is useful since there is no way of knowing for certain if an event was produced by $t\bar{t}$ or something else. The likelihood of a top pair decaying to the jets and lepton we measured is calculated using the following equation.

$$\begin{aligned}
L = & B(m_{q_1 q_2 q_3} | m_{top}, \Gamma_{top}) \cdot \exp(-4 \cdot \ln 2 \cdot \frac{(m_{q_1 q_2} - m_W)^2}{\Gamma_W^2}) \\
& \cdot B(m_{q_4 l \nu} | m_{top}, \Gamma_{top}) \cdot B(m_{l \nu} | m_W, \Gamma_W) \\
& \prod_{i=1}^4 W_{jet}(E_{jet,i}^{meas} | E_{quark,i}) \cdot W_l(E_l^{meas} | E_l) \\
& \cdot W_{miss}(E_x^{miss} | p_x^\nu) \cdot W_{miss}(E_y^{miss} | p_y^\nu)
\end{aligned}$$

Where $B(m|M, \Gamma)$ is a Breit Wigner distribution which gives the probability that a particle with mass M and width Γ will decay to particles with a center of mass energy m , and $W(E^{meas} | E^{truth})$ is a transfer function which gives the probability that our detector measures a jet with energy E^{meas} given it was produced by a quark with energy E^{truth} . Due to different responses in different regions of the detector, there are separate transfer functions for given η ranges. The transfer functions must be included in this calculation to account for the effects of hadronization and the detector. The E_{quark} , p^ν and E_l parameters are chosen to maximize the likelihood L . The permutation of jets that maximizes the likelihood is then used to reconstruct the quarks and W bosons in the underlying event. It should be noted that while this technique of kinematic fitting was developed specifically for $t\bar{t}$ events, it can be applied to other types of events.

4 Transfer Functions

4.1 Monte Carlo Sample and Event Selection

We computed the 13 TeV transfer functions using a Monte Carlo simulation of $t\bar{t}$ events. Figure 6 displays one of the Feynman diagrams for this process. The $t\bar{t}$ sample was generated with POWHEG-BOX v2 using CT10 PDF interfaced to PYTHIA 6.428 for parton shower, using the PERUGIA2012 tune with CTEQ6L1 PDF for the underlying event descriptions. EVTGEN v1.2.0 is used for properties of the bottom and charm hadron decays. The mass of the top quark is set to $m_t = 172.5$ GeV. At least one top quark in the $t\bar{t}$ event is required to decay to a final state with a lepton. The parameter HDAMP, used to regulate the high- p_T radiation in POWHEG, is set to m_t for good data/MC agreement in the high p_T region.

The following selection cuts were then applied to the $t\bar{t}$ sample. First, we required that there be at least four jets in the event and at least two of them must be b-tagged ($MVc20 > -0.7887$). Furthermore, each jet must have $p_T > 20$ GeV and $|\eta| < 2.5$. We also require that there is a single lepton in the event and it has $p_T > 20$ GeV and $|\eta| < 2.5$. To ensure the presence of a neutrino we require that $MET > 20$ GeV.

The $t\bar{t}$ sample that was produced stored both truth and reconstruction information. In order to compute the transfer functions, we needed to matched each truth level quark to a reconstructed jet. A quark and jet were matched if $\Delta R(quark, jet) < 0.3$. We also required that each quark was matched uniquely to a jet. This matching was also required to be unique, ie. each quark matched to exactly one jet and vice versa. Leptons were matched according to the requirement $\Delta R(lepton_{truth}, lepton_{reco}) < 0.1$. There was no matching requirement applied to match MET to the neutrino.

4.2 Local Fit

The 8 TeV transfer functions that were previously used were double Gaussians whose parameters were functions of E_{truth} . We decided to use the same parametrization for the 13 TeV transfer functions; the exact parametrizations used are in the appendix.

The first step of calculating the 13 TeV transfer functions is a fit to the Monte Carlo data. The data from the Monte Carlo was saved in the form $(\frac{E_{truth} - E_{reco}}{E_{truth}}, E_{truth})$ and then each

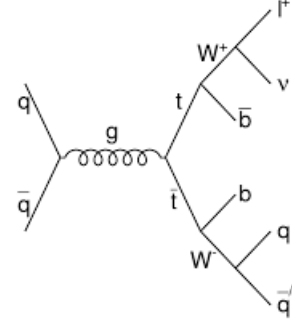


Figure 6:
 $t\bar{t}$ Feynman Diagram ⁵

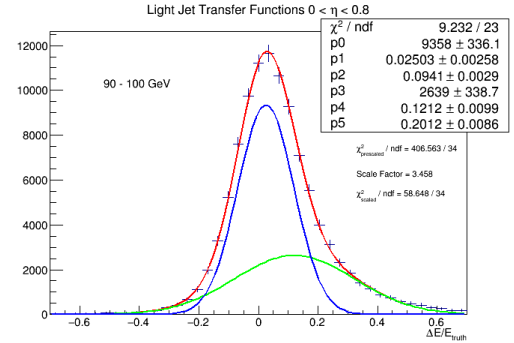


Figure 7: A typical local fit. The red is the double Gaussian fit, the blue is the larger Gaussian and the green is the smaller Gaussian.

⁵Photo Credit: InspireHEP.net

10 GeV bin of E_{truth} was fit to a double Gaussian according to the following algorithm. First, the peak of the distribution was fit to a single Gaussian. Then the parameters from that fit were used to initialize the parameters in the double Gaussian fit. Due to issues with convergence, at this point in the process the errors were scaled up so the χ^2 / dof was equal to 1. Then the fit was run again. This was able to fix the issues with the fit converging, allowing us to have a description of the transfer functions at fixed energies in terms of double Gaussians. Figure 7 displays the result of one of these local fits for light jets that came from a quark with energy in the range 90 – 100 GeV.

4.3 Global Fit

To apply the transfer functions to the kinematic likelihood method, we need to be able to interpolate over the entire energy range, not just know the values at some discrete set of energies. To do this interpolation, we first fit the parameters of the double Gaussian fits from the previous section, the two means, standard deviations and scale factor, to functions of E_{truth} . The exact functions chose depended on the type of particle, and are in the appendix.

The parameters from this fit are then used as the starting point of a global fit to optimize the parameters. The algorithm used for the global fit is based off of Bayes theorem $P(\text{model}|\text{data}) \propto P(\text{data}|\text{model})$ where in this fit the data comes from the Monte Carlo sample and the model is the choice of fit parameters. Assuming the number of entries in a given histogram bin is described by a Poisson distribution, $\log(P(\text{data}|\text{model}))$ can be approximated by

$$\sum_{bins} n_{data} \log\left(\frac{n_{model}}{n_{data}}\right) + n_{data} - n_{model}$$

where n_{data} is the number of entries from the Monte Carlo sample and n_{model} is the number of entries predicted by the parameters in question.

The optimal fit will be the choice of parameters that maximizes this probability. We utilized the Metropolis algorithm to solve this optimization problem. The Metropolis algorithm works by generating a set of candidate parameters from some set of prior distributions. For this implementation of the algorithm, we used a flat prior distributions centered on the parameters from the previous step of the fit. Then if the conditional probability $P(\text{data}|\text{candidate})$ is larger than the conditional probability for the previous set of parameters, we replace the previous set of parameters with the candidate parameters. If this is not the case, then a random number r is generated from the range $(0, 1)$ and if $\frac{P(\text{data}|\text{candidate})}{P(\text{data}|\text{previous})} > r$ the candidate parameters replace the previous set of parameters. Otherwise, we will just keep the previous set of parameters. By iterating this process, it will eventually converge on the parameters that maximize this probability.

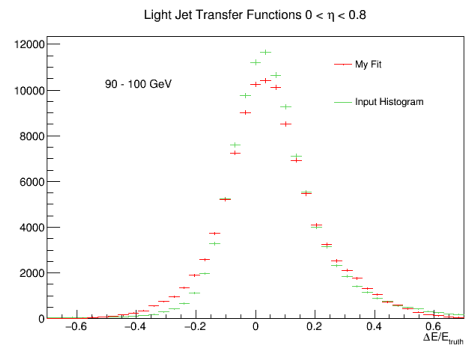


Figure 8: Result of the Global Fit.

5 Results

Once the transfer functions were computed, they were applied in an analysis of the Monte Carlo simulation of $t\bar{t}$ production different from the one used to calculate them. Figure 9 shows the reconstructed mass distribution of the two top quarks in the event both without using kinematic fitting and using kinematic fitting. The resolution of the top mass can be calculated by fitting these distributions to a Gaussian. The results of this fit are displayed in the below table.

m_t Resolution	Fitted	Reconstructed
Hadronic	11.918(3) GeV	24.648(8) GeV
Leptonic	4.967(2) GeV	10.798(3) GeV

As the table shows, the kinematic likelihood method is able to greatly reduce the uncertainties introduced by the detector and hadronization.

The transfer functions computed here have been approved by ATLAS and will be implemented in the KLFitter toolkit that performs kinematic fitting. With this approval, the transfer functions computed here will be applied to other analyses of ATLAS data.

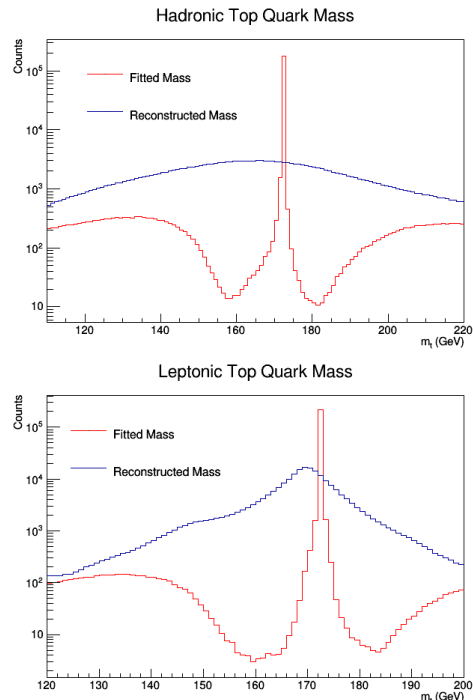


Figure 9: Top Mass Distributions

References

- [1] Griffiths, D. J. (2014). *Introduction to Elementary Particles*. Weinheim: Wiley-VCH Verlag.
- [2] Schwartz, M. D. (2014). *Quantum Field Theory and the Standard Model*. New York: Cambridge University Press.
- [3] Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC - The ATLAS Collaboration. Phys.Lett. B716 (2012) 1-29 arXiv:1207.7214 [hep-ex]
- [4] A likelihood-based reconstruction algorithm for top-quark pairs and the KLFitter framework - Erdmann, Johannes et al. Nucl.Instrum.Meth. A748 (2014) 18-25 arXiv:1312.5595 [hep-ex]

6 Appendix

6.1 Light Jet Transfer Functions

The functional form used for the light jet transfer functions was

$$\frac{1}{\sqrt{2\pi}(\sigma_1 + p_3\sigma_2)} \left(e^{-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2} + p_3 e^{-\frac{1}{2}\left(\frac{x-\mu_2}{\sigma_2}\right)^2} \right)$$

where $x = \frac{E_{truth}-E_{reco}}{E_{truth}}$. Note that all energies are measured in units of GeV. The parametrizations used for the double Gaussian parameters are

$$\mu_1 = a_1 + \frac{b_1}{E_{truth}}$$

$$\sigma_1 = a_2 + \frac{b_2}{\sqrt{E_{truth}}}$$

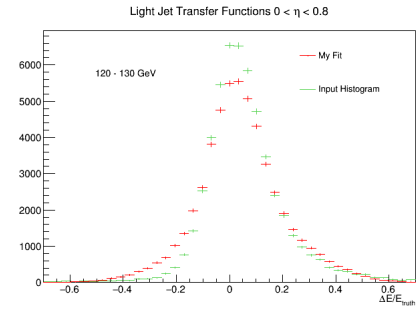
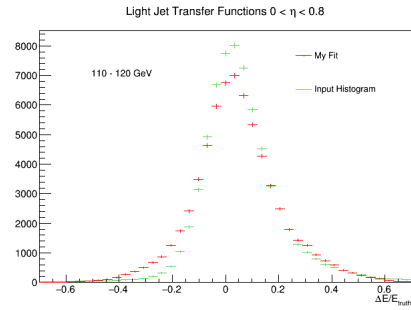
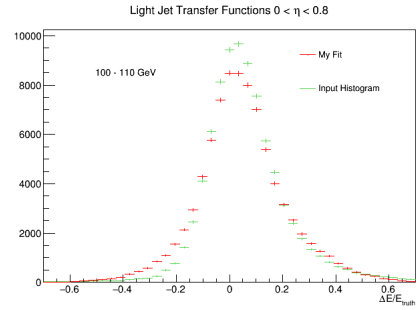
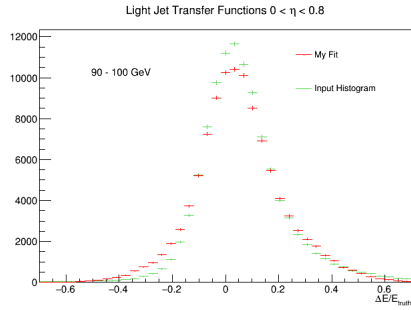
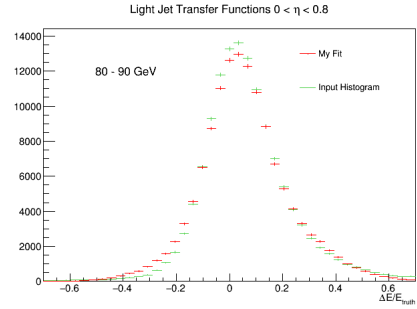
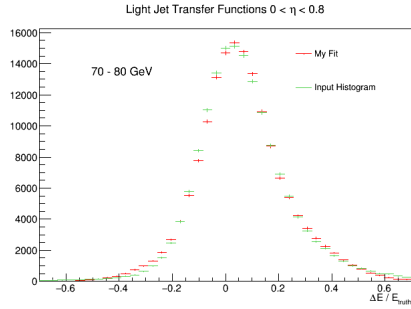
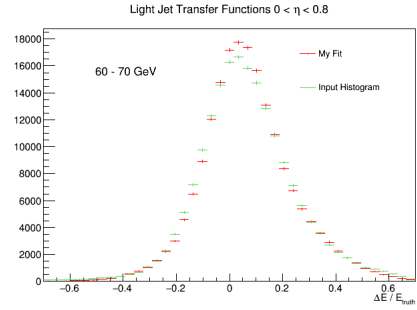
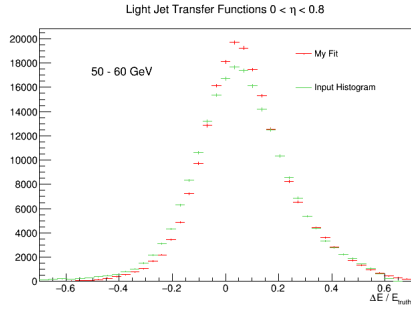
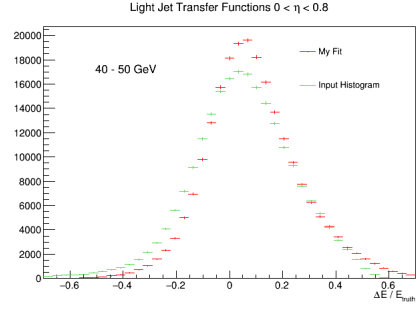
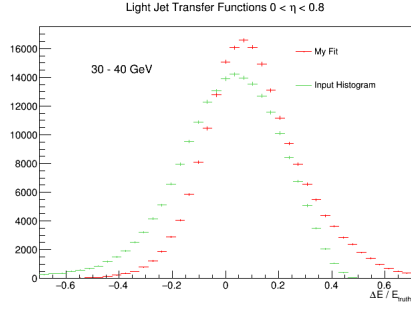
$$p_3 = a_3 + \frac{b_3}{E_{truth}}$$

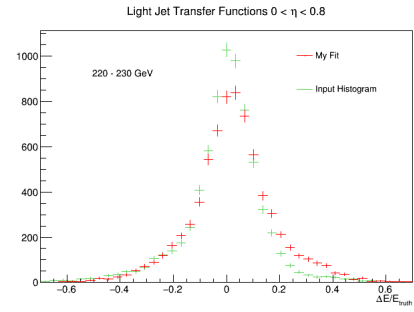
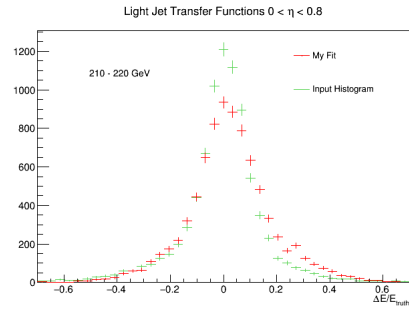
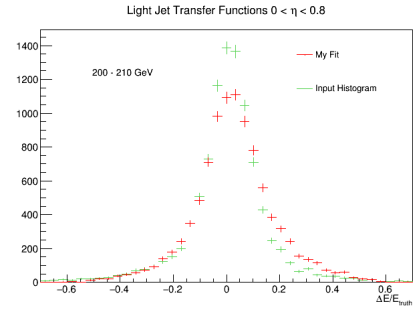
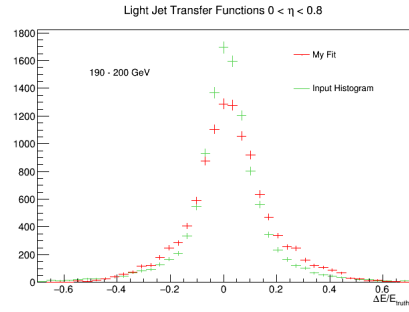
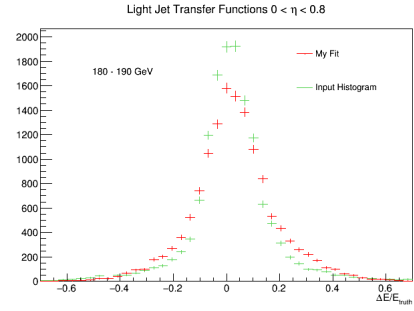
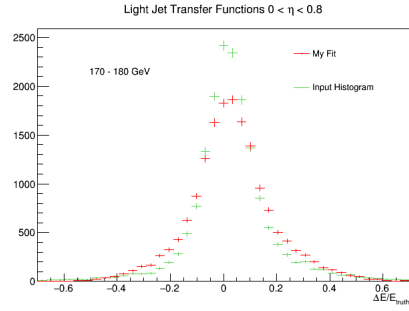
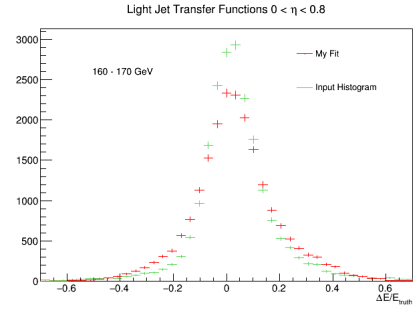
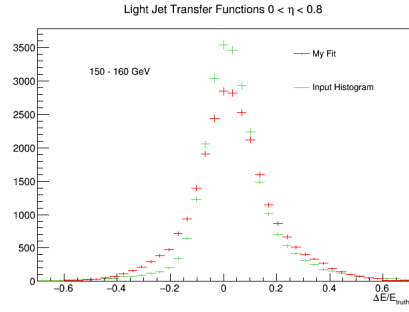
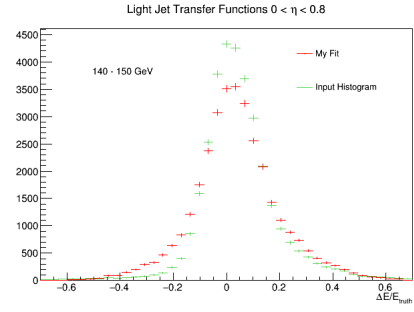
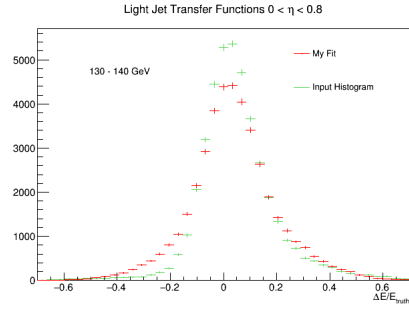
$$\mu_2 = a_4 + \frac{b_4}{\sqrt{E_{truth}}}$$

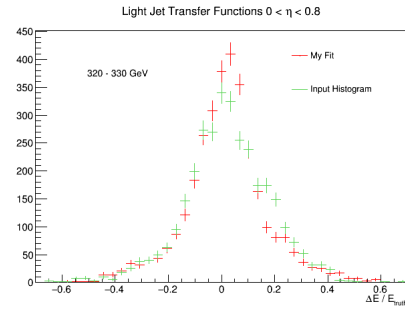
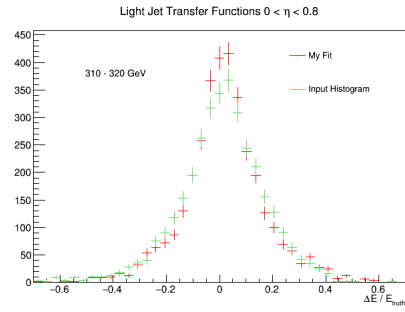
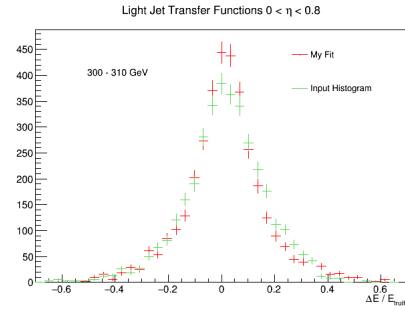
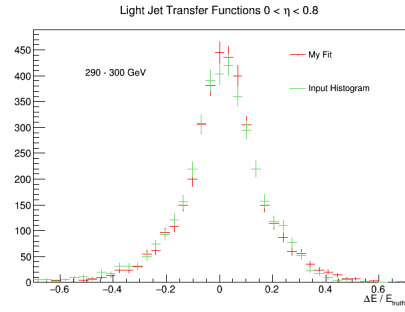
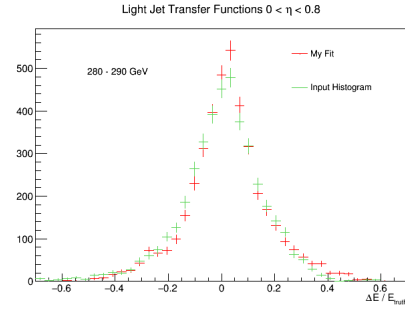
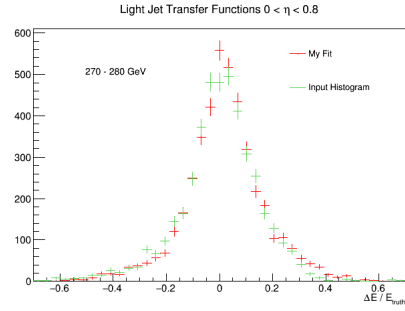
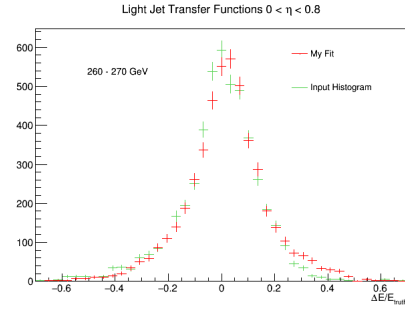
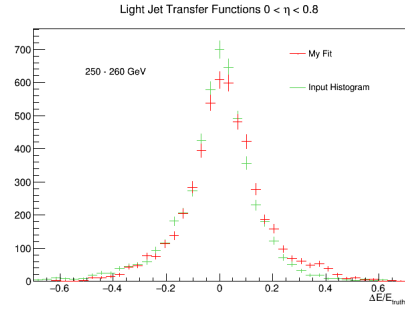
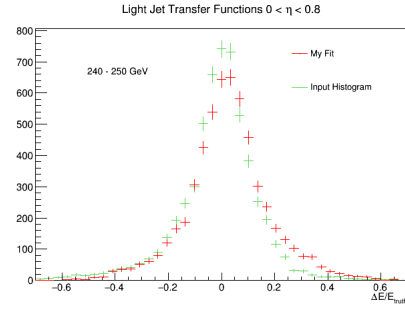
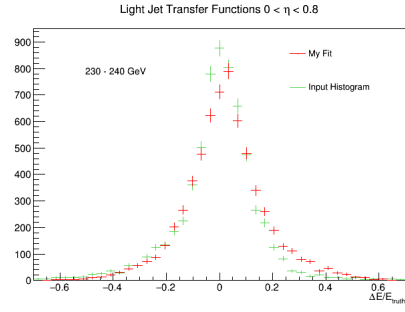
$$\sigma_2 = a_5 + b_5 E_{truth}$$

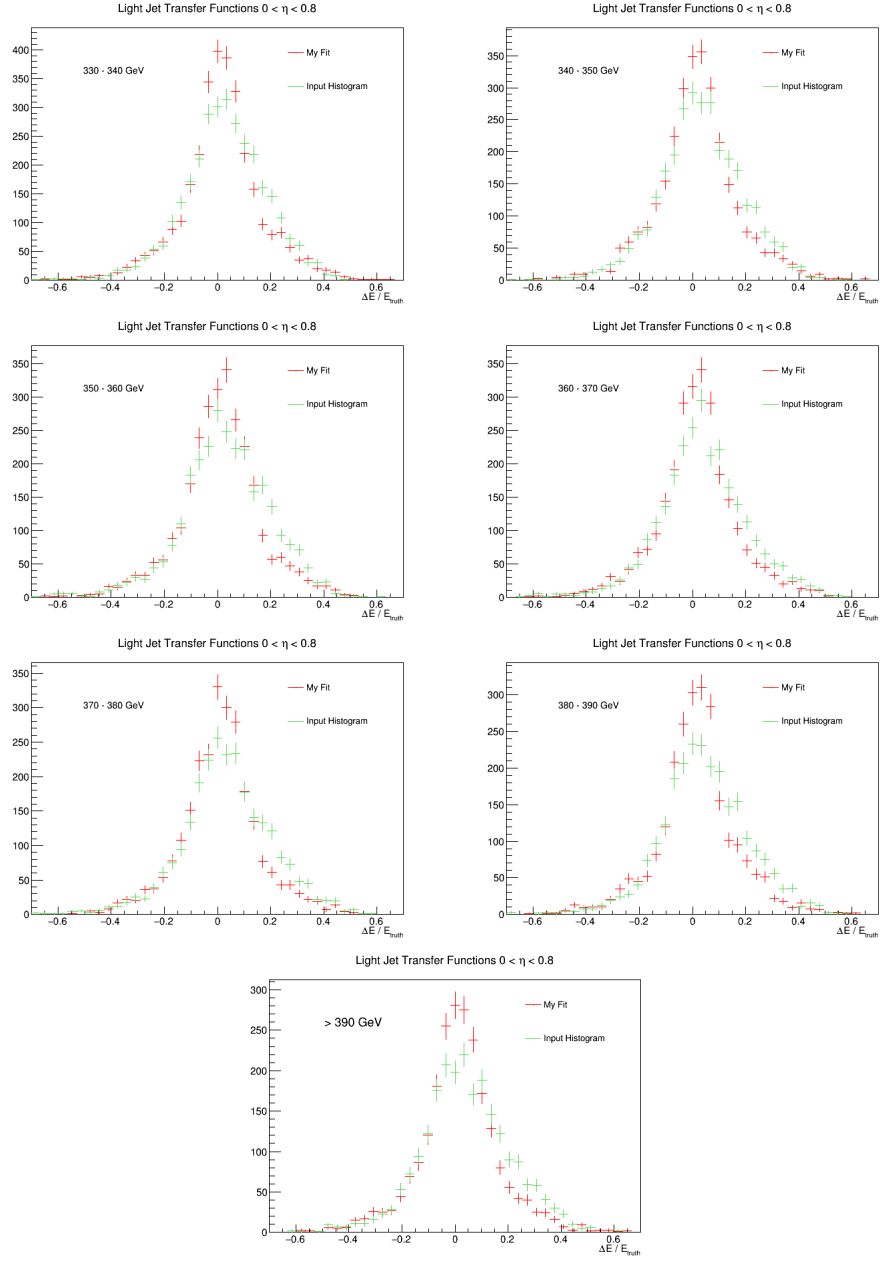
Eta Region 1

Transfer Function Parameters	
a_1	0.0120801
b_1	1.22818
a_2	0.0489807
b_2	0.418622
a_3	0.409053
b_3	10.2044
a_4	-0.0556186
b_4	1.26076
a_5	0.215323
b_5	$-5.49251 \cdot 10^{-5}$



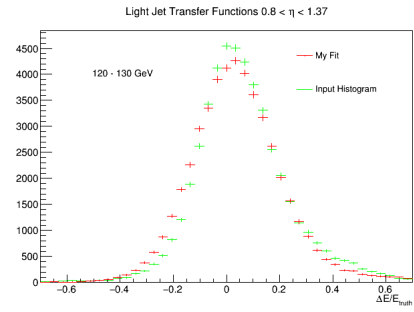
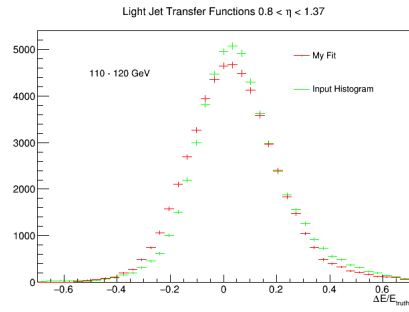
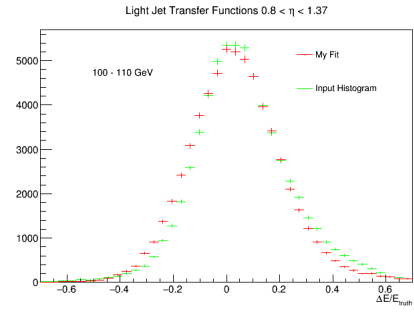
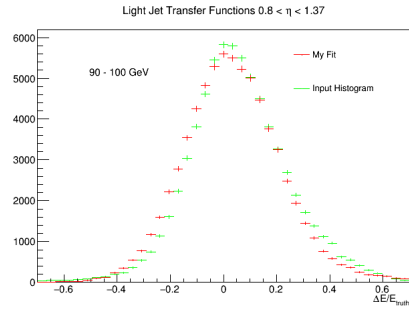
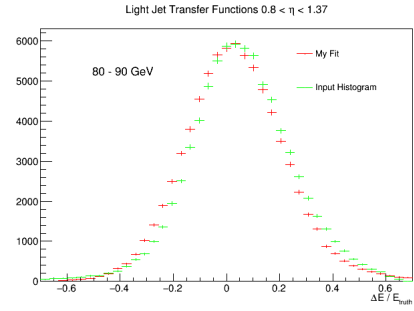
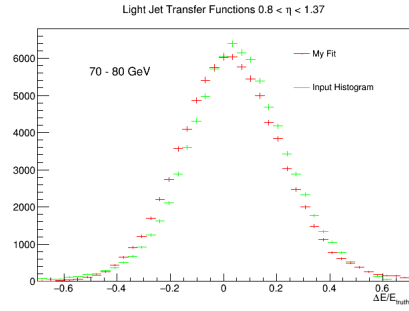
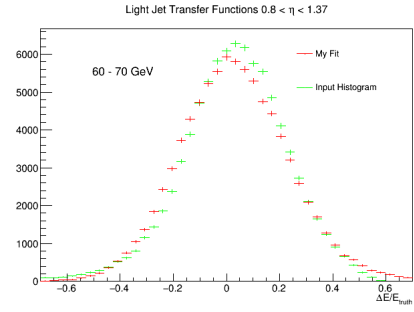
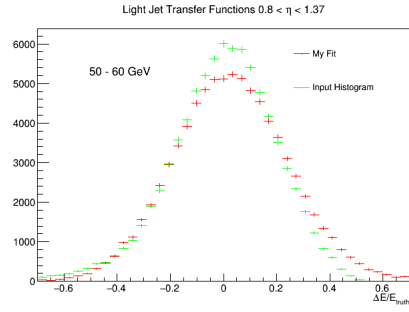
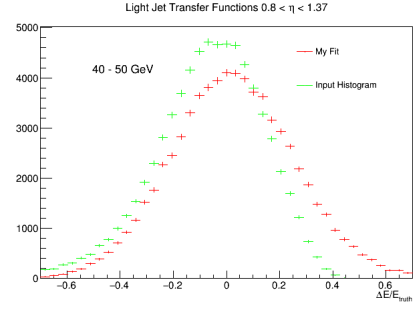
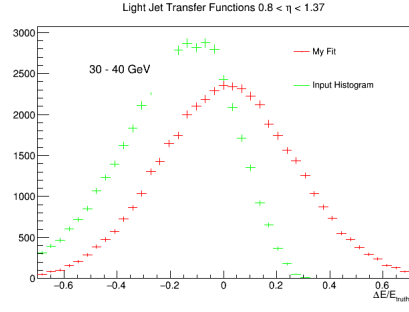


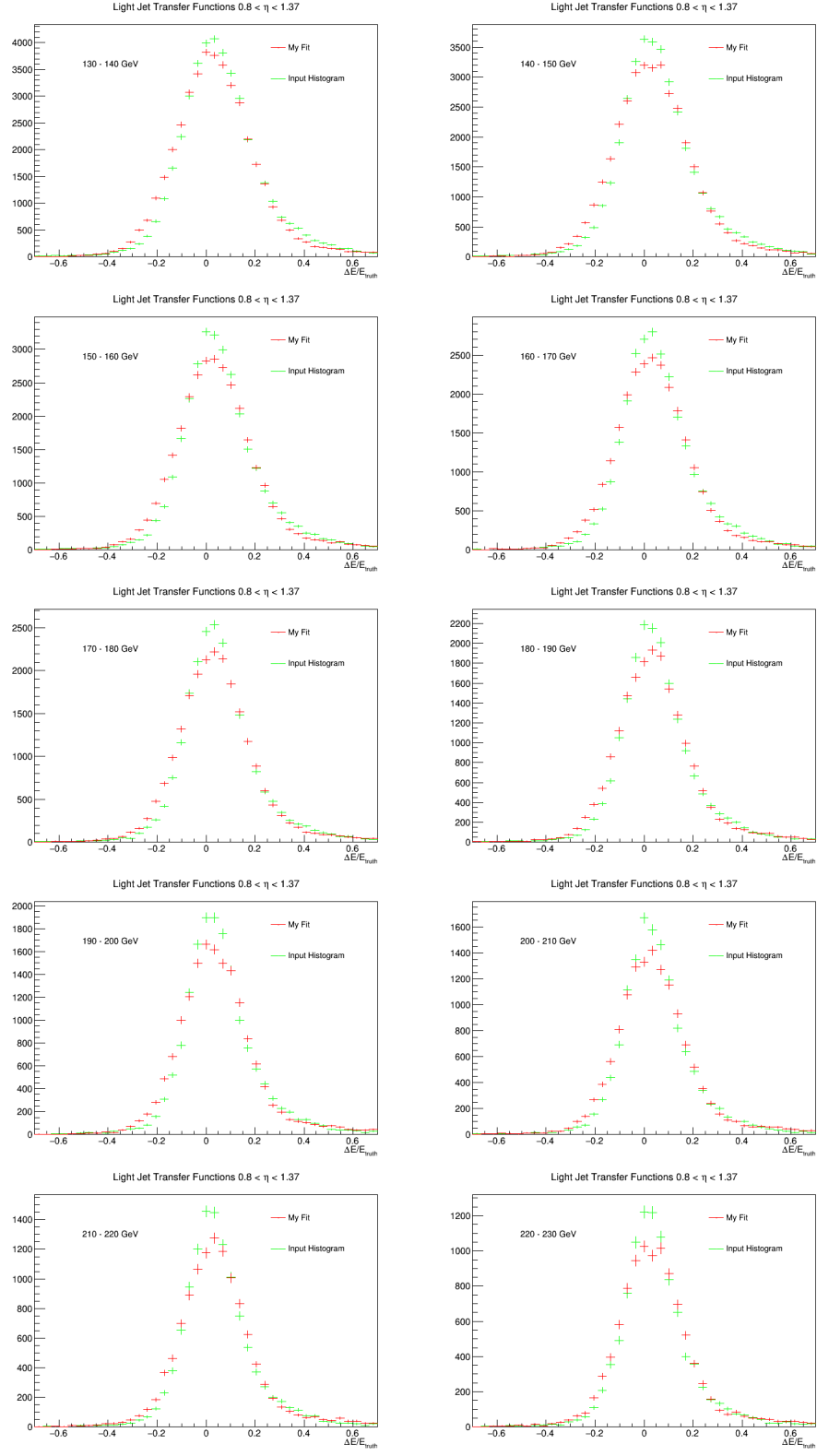


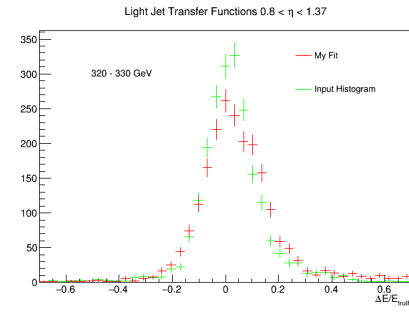
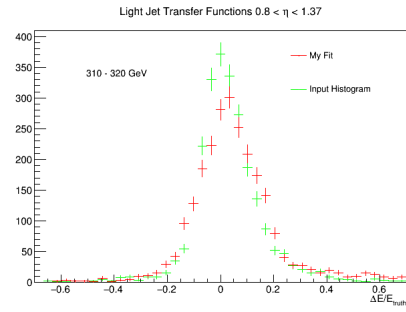
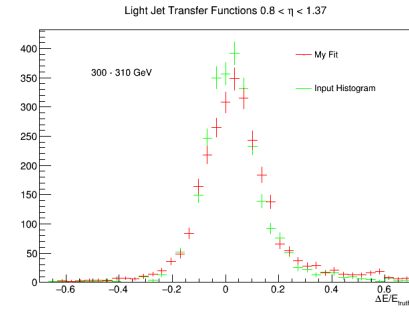
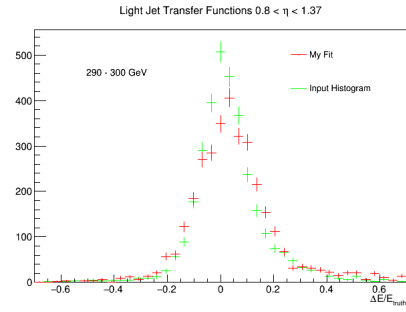
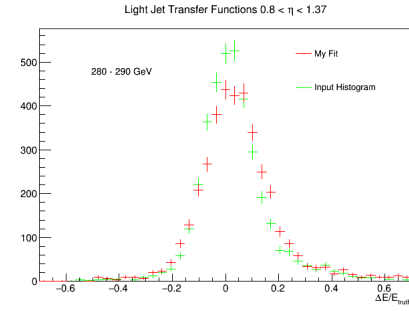
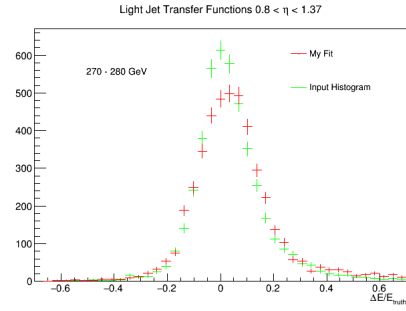
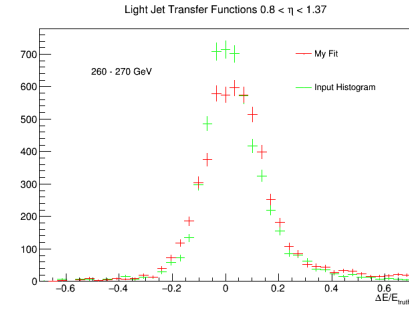
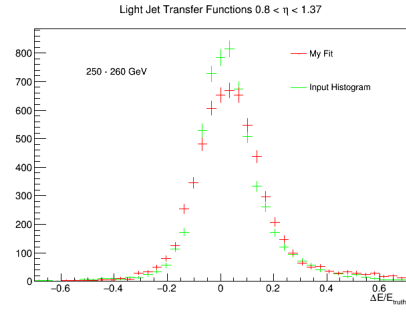
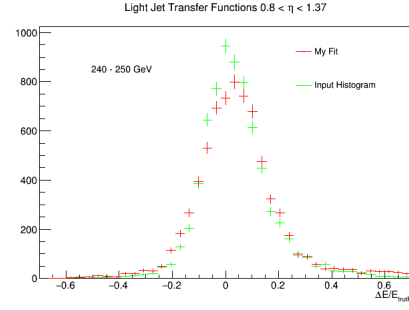
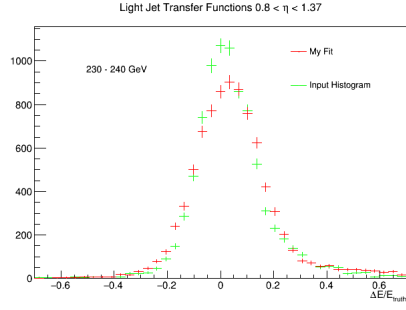


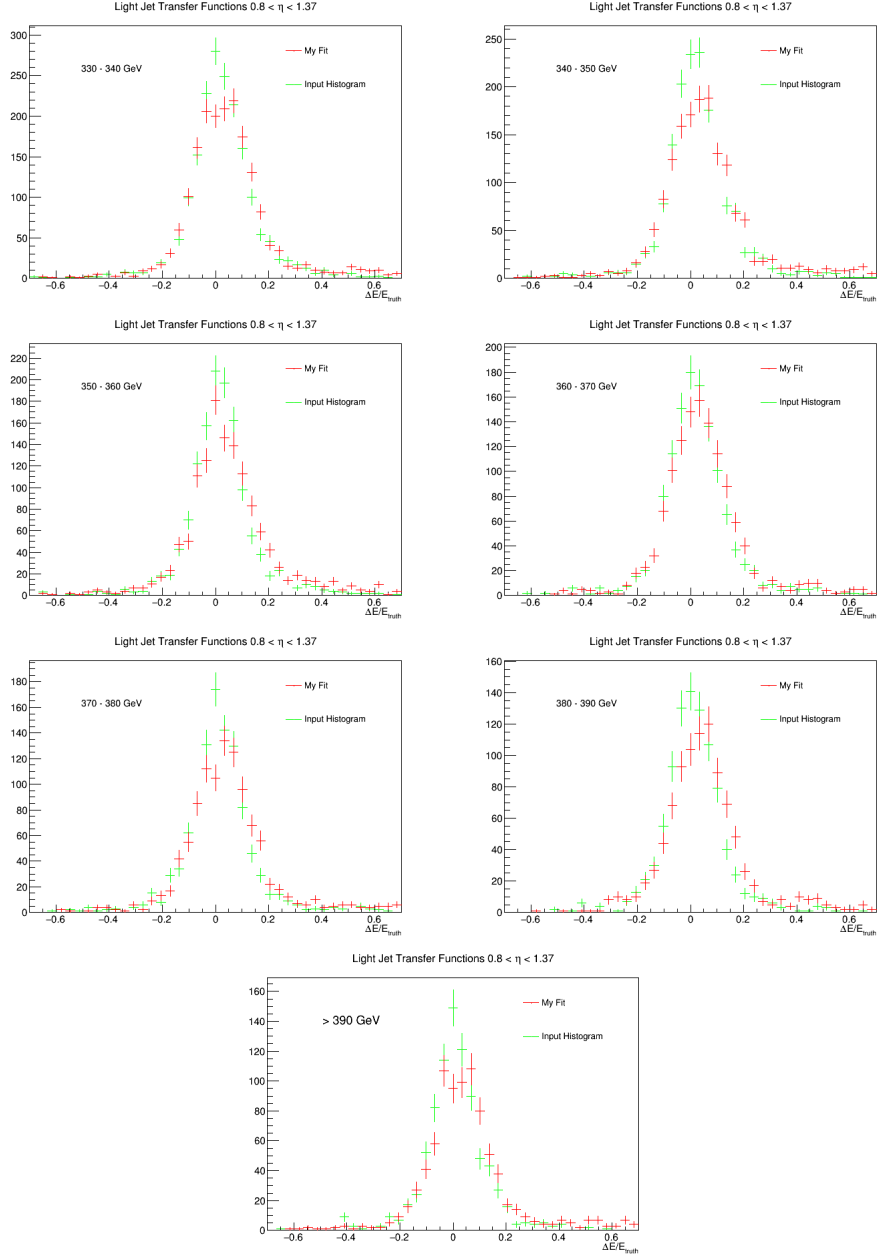
Eta Region 2

Transfer Function Parameters	
a_1	0.0287369
b_1	-0.783209
a_2	0.0225738
b_2	1.36549
a_3	0.0621159
b_3	1.05336
a_4	0.238389
b_4	-0.601056
a_5	0.273594
b_5	0.00024647



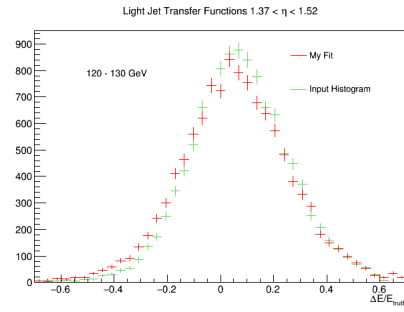
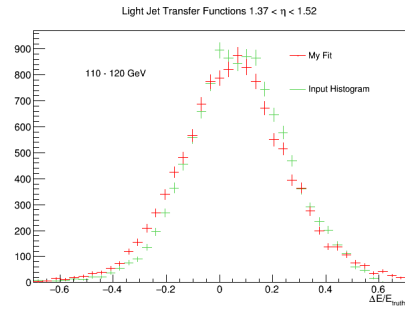
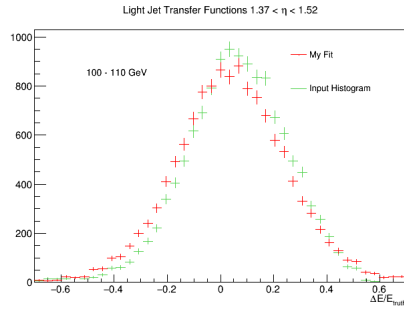
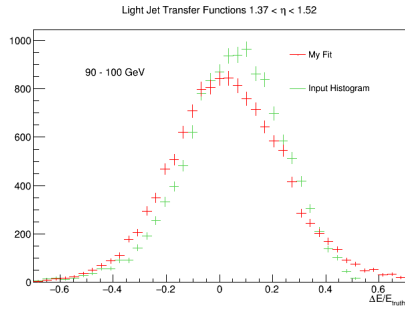
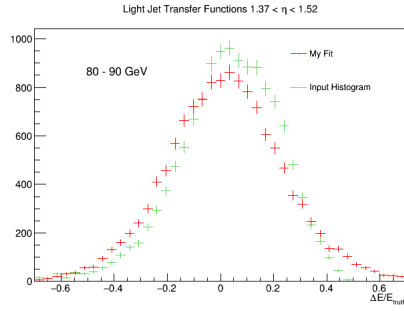
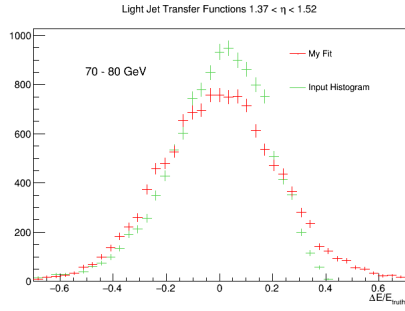
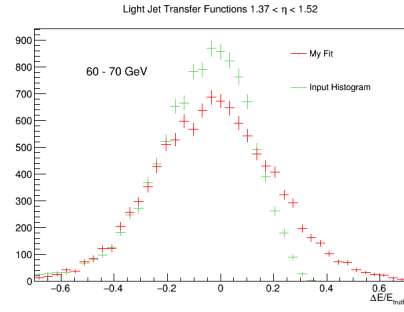
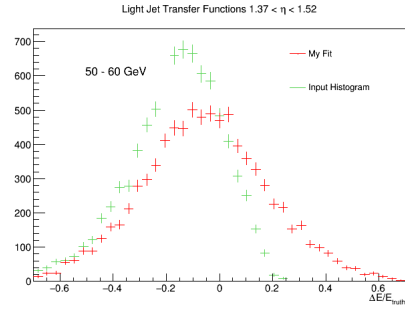
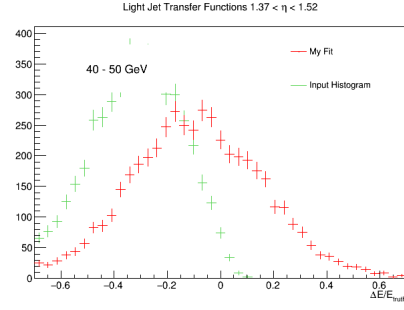
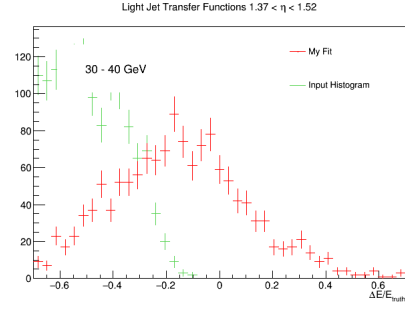


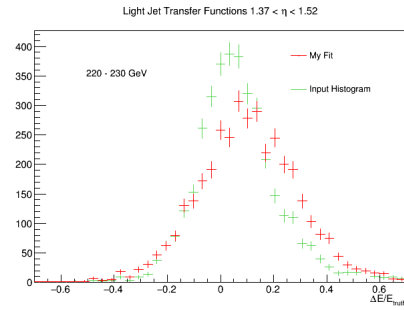
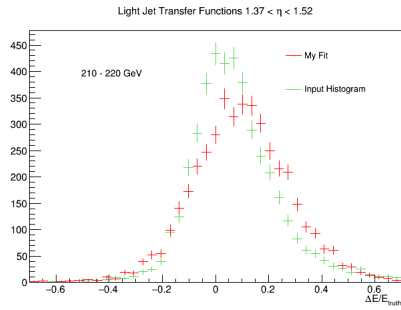
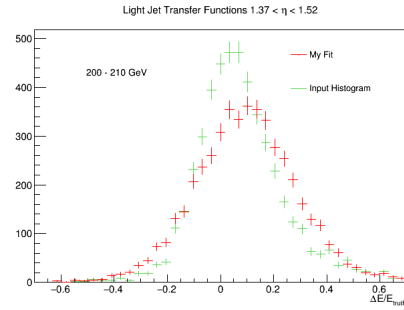
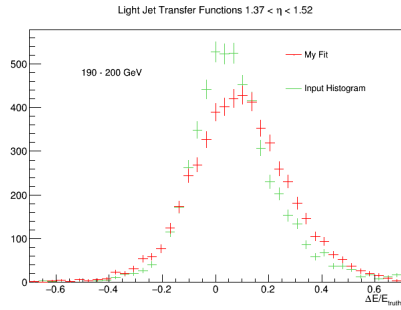
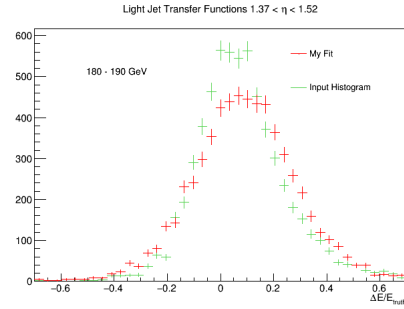
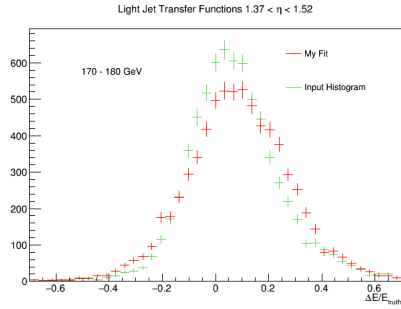
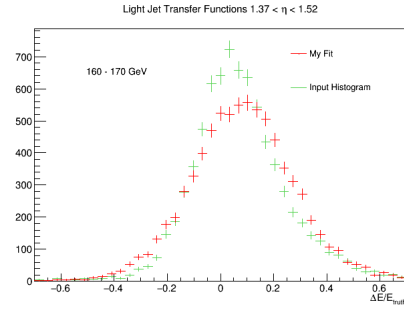
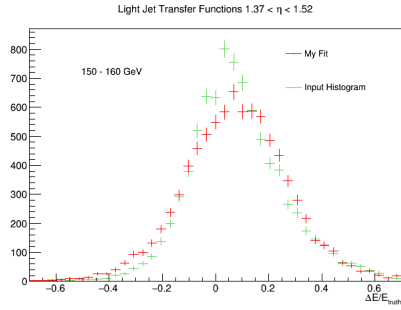
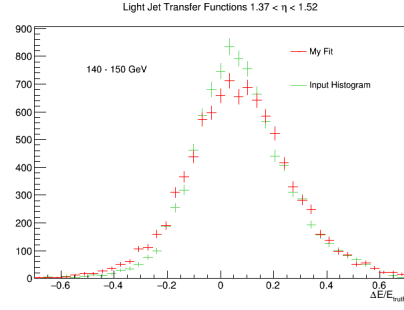
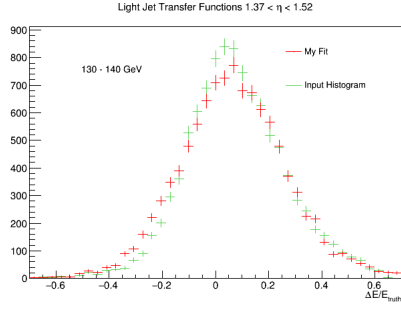


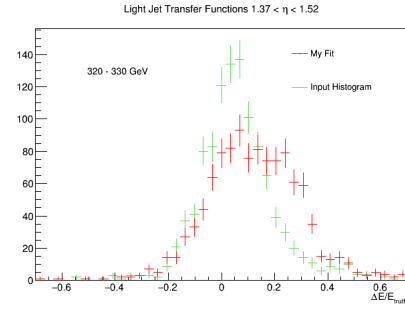
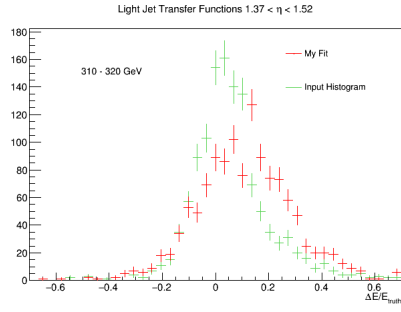
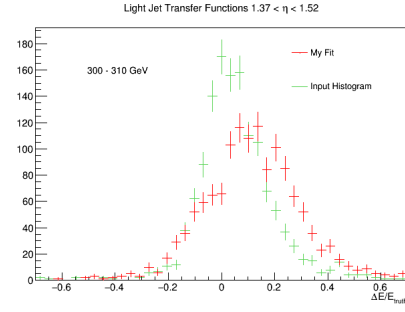
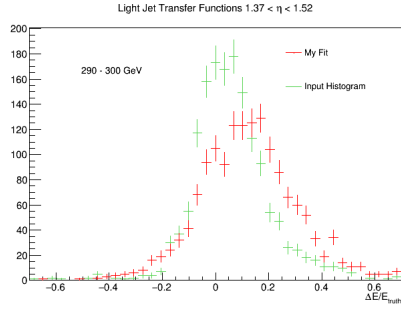
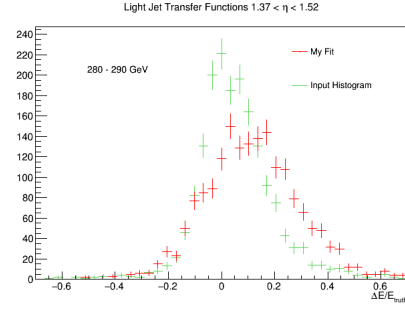
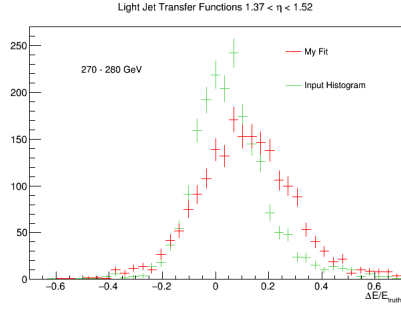
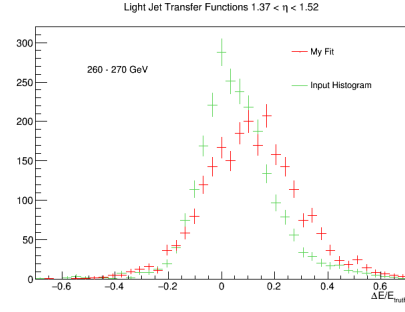
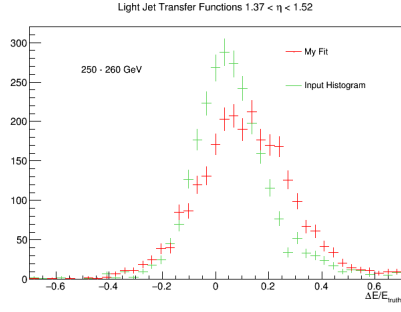
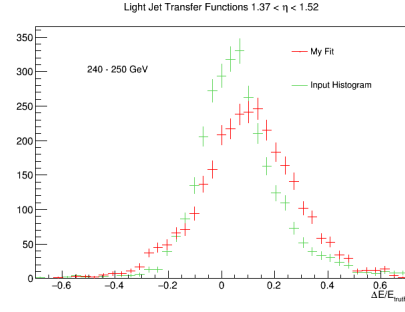
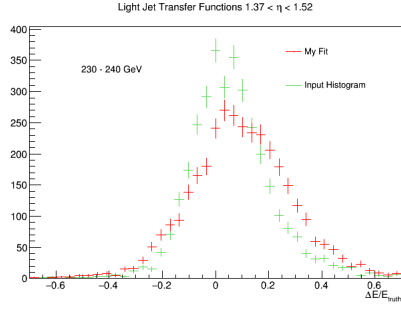


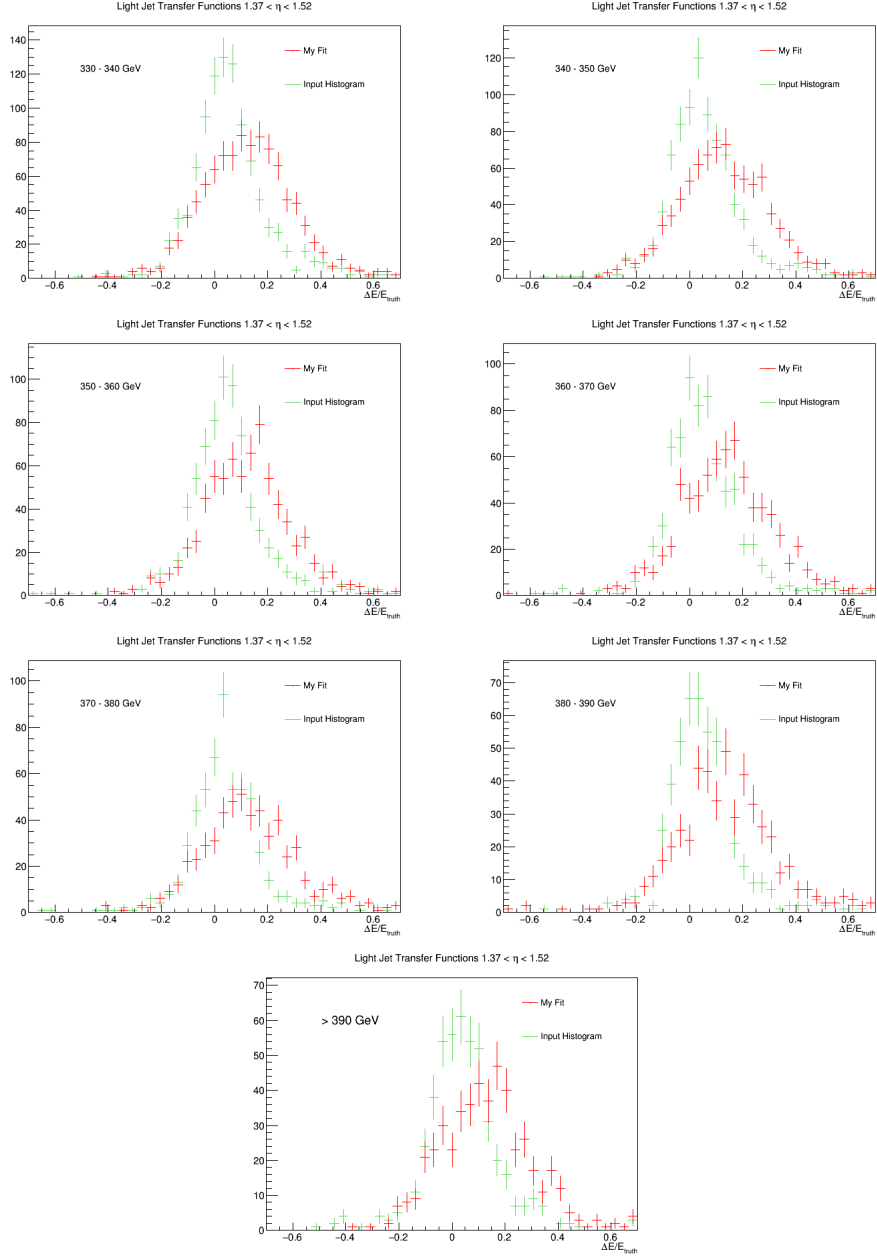
Eta Region 3

Transfer Function Parameters	
a_1	0.140728
b_1	-10.9913
a_2	0.100472
b_2	0.855606
a_3	0.115908
b_3	8.96468
a_4	0.338261
b_4	-2.77732
a_5	0.310263
b_5	$-9.96212 \cdot 10^{-5}$



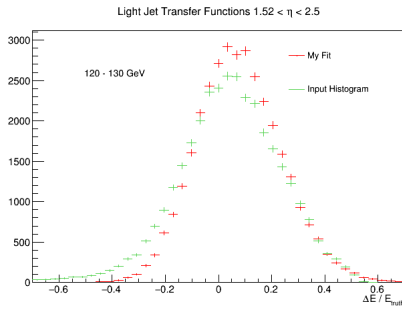
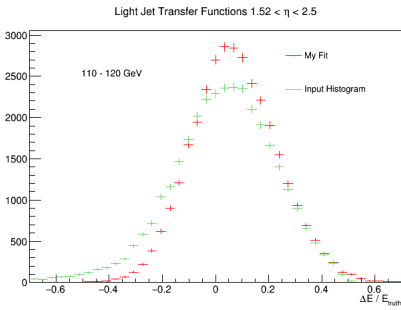
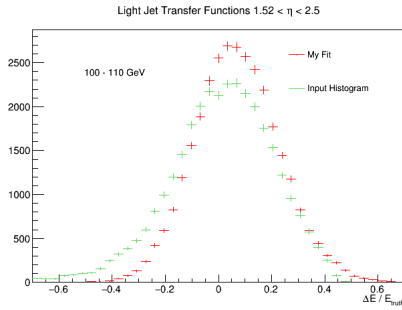
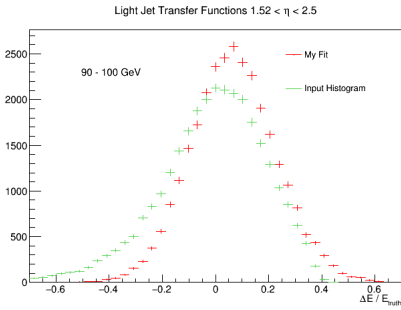
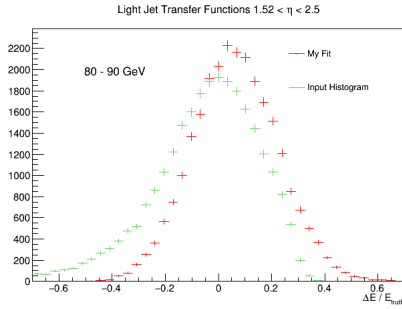
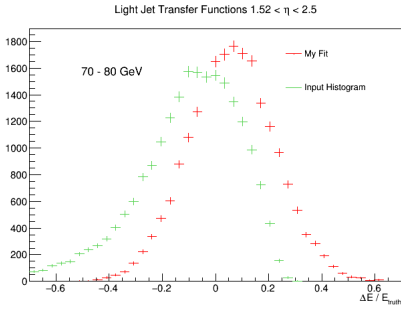
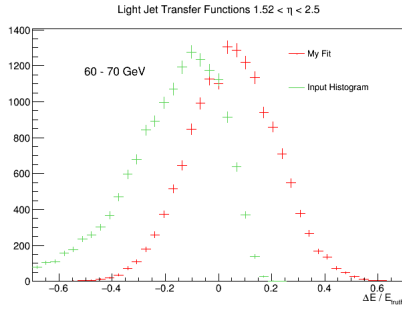
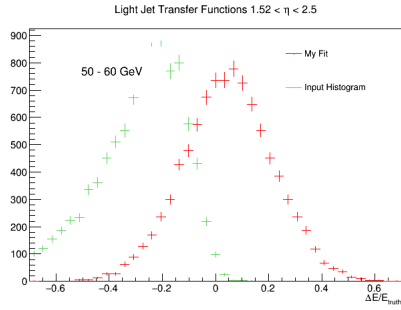
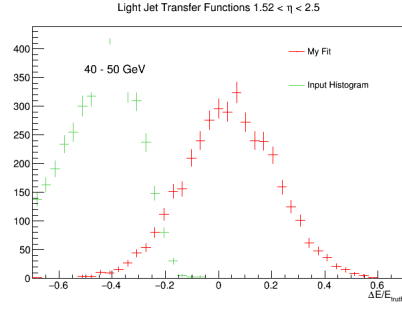
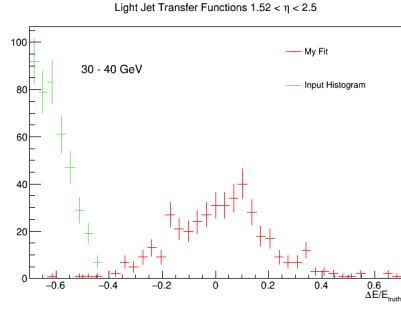


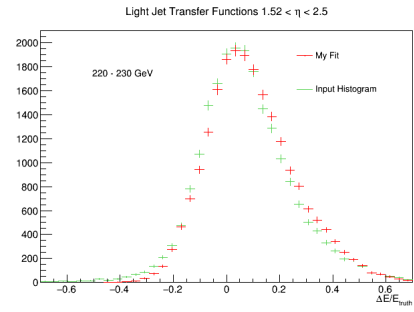
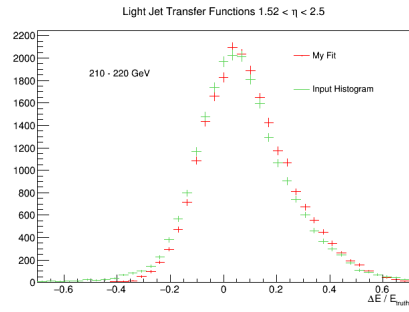
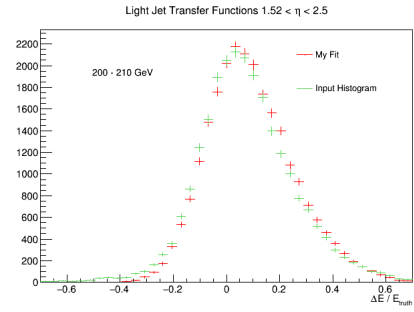
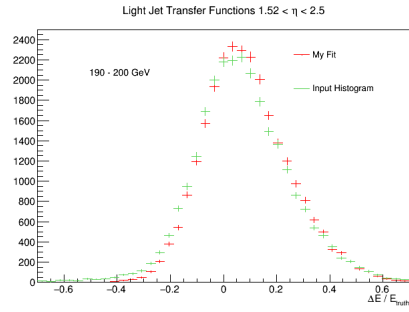
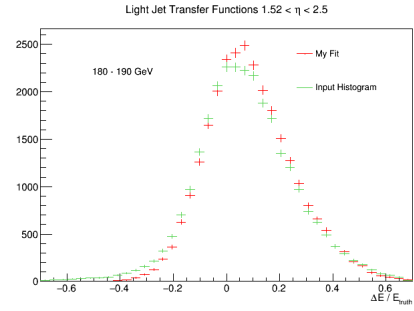
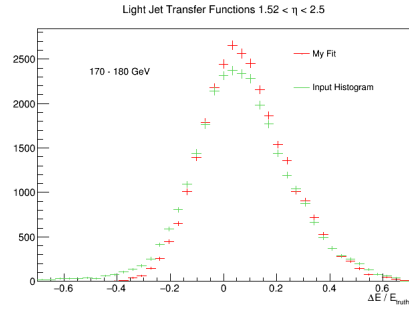
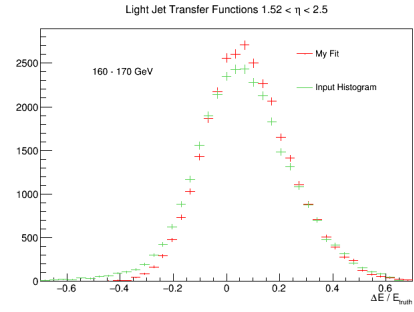
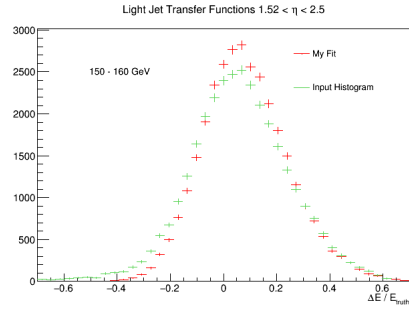
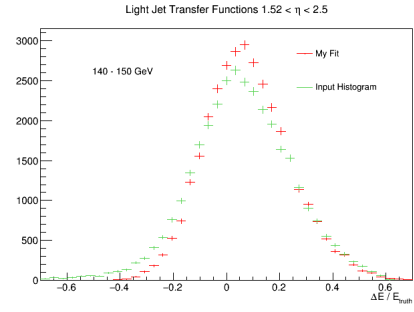
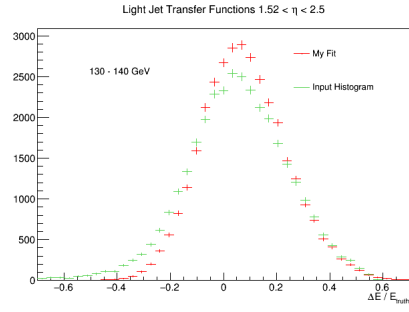


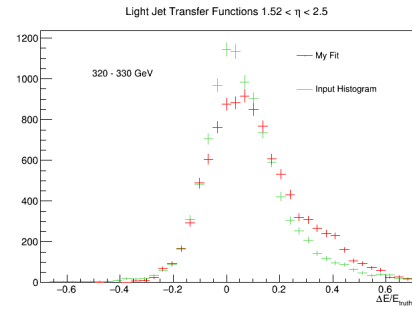
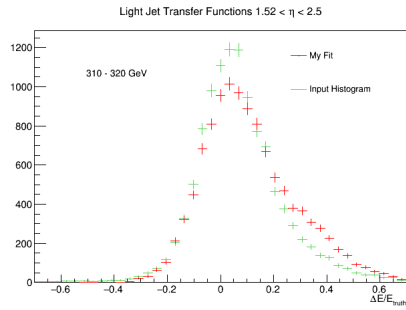
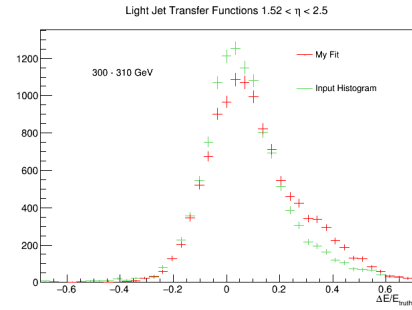
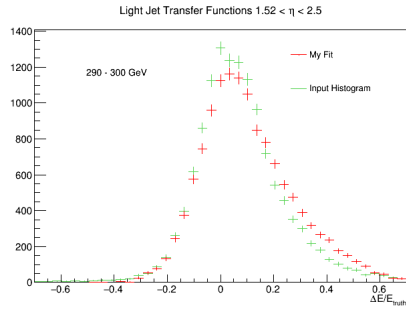
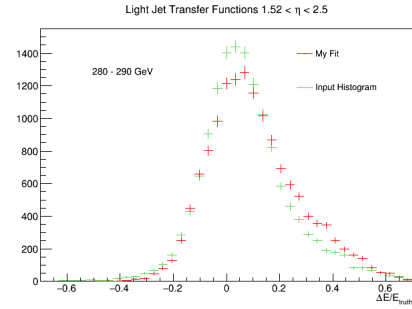
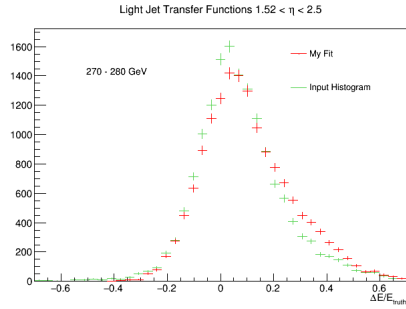
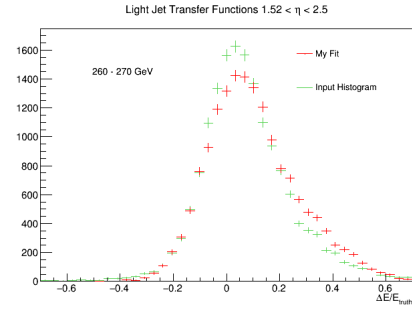
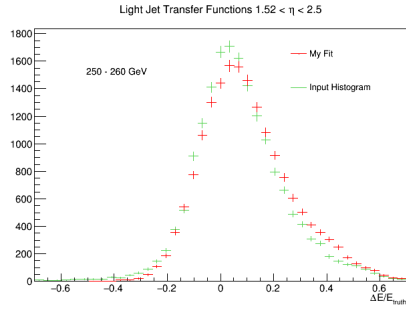
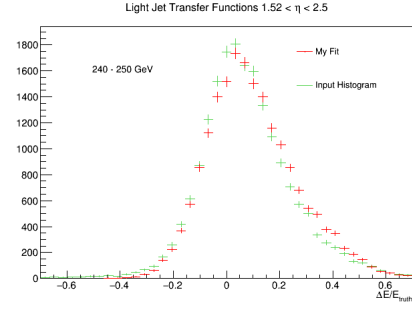
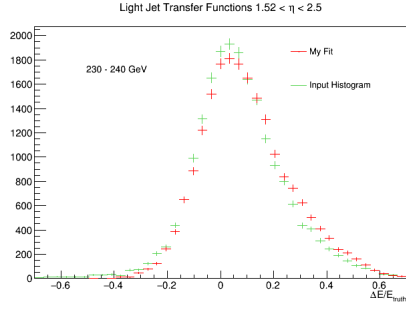


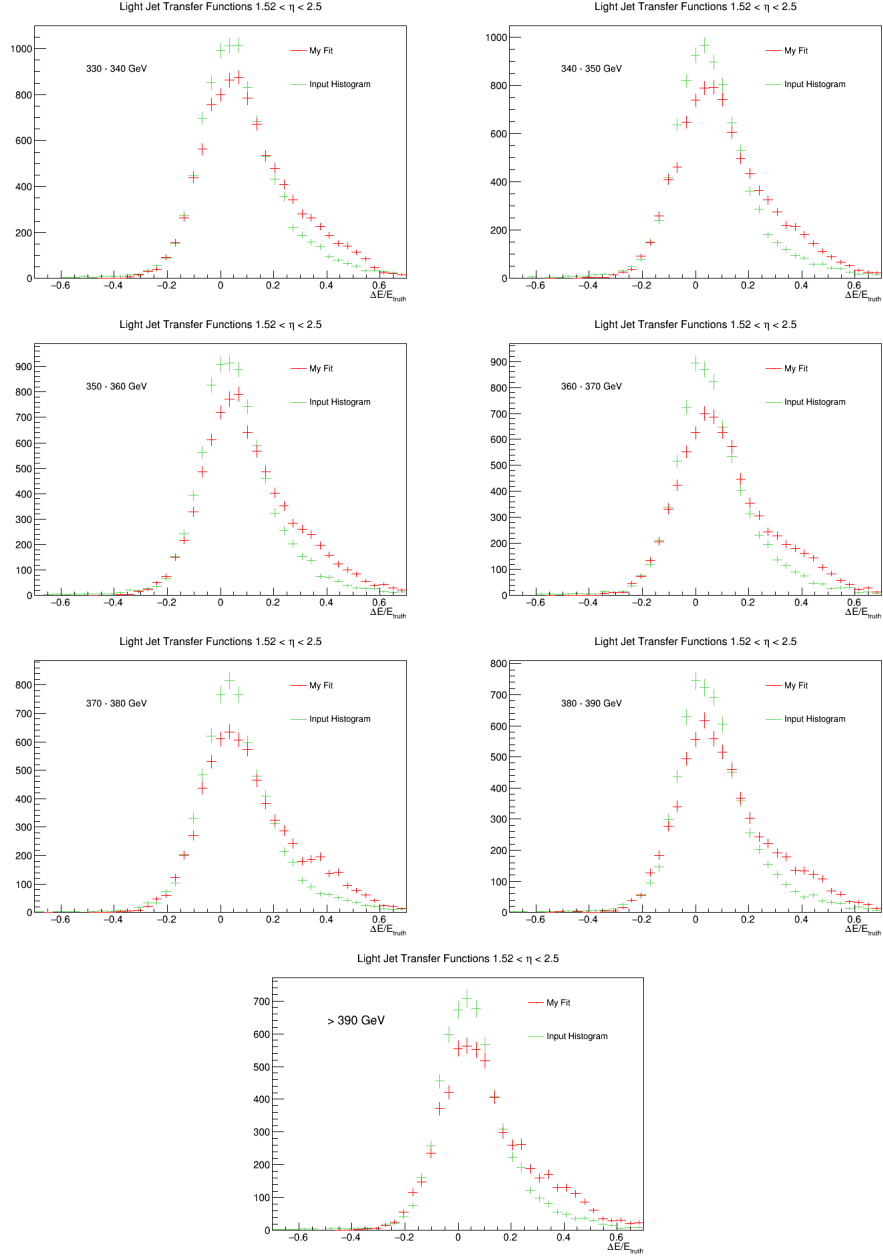
Eta Region 4

Transfer Function Parameters	
a_1	0.0239971
b_1	0.317509
a_2	0.063023
b_2	0.808414
a_3	0.406621
b_3	11.8981
a_4	0.314448
b_4	-1.73275
a_5	0.148917
b_5	0.000125134









6.2 B Jet Transfer Functions

The functional form used for the b jet transfer functions was

$$\frac{1}{\sqrt{2\pi}(\sigma_1 + p_3\sigma_2)} \left(e^{-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2} + p_3 e^{-\frac{1}{2}\left(\frac{x-\mu_2}{\sigma_2}\right)^2} \right)$$

where $x = \frac{E_{truth}-E_{reco}}{E_{truth}}$. Note that all energies are measured in units of GeV. The parametrizations used for the double Gaussian parameters are

$$\mu_1 = a_1 + \frac{b_1}{E_{truth}}$$

$$\sigma_1 = a_2 + \frac{b_2}{\sqrt{E_{truth}}}$$

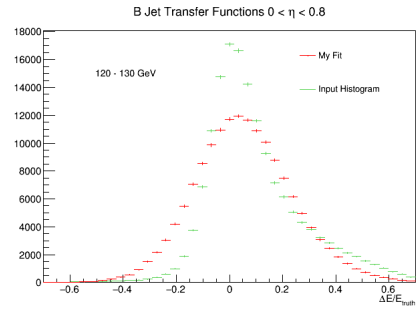
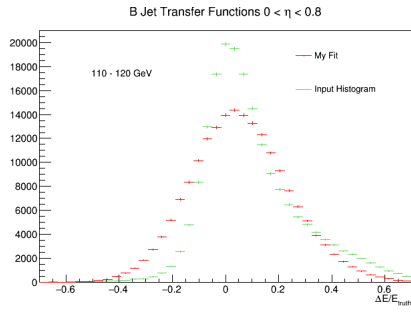
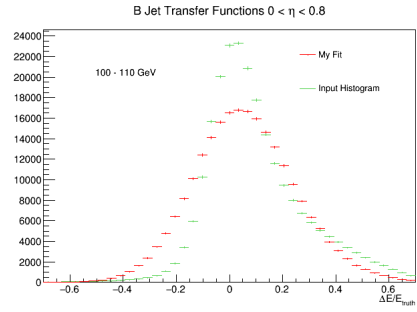
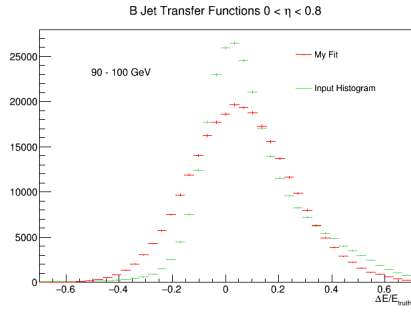
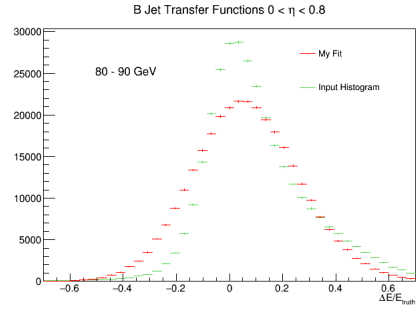
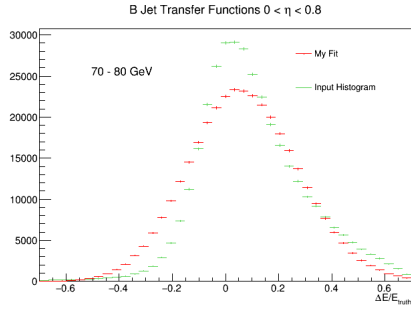
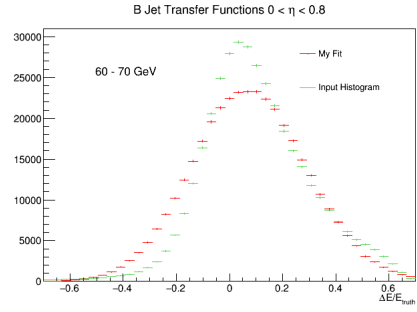
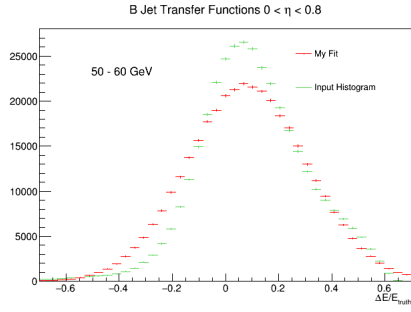
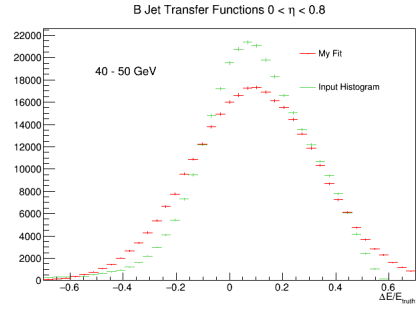
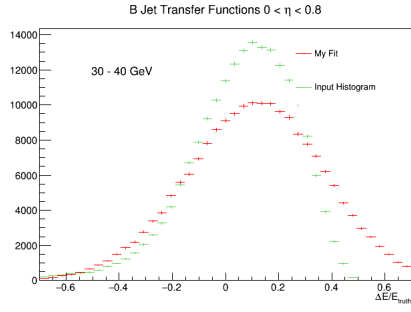
$$p_3 = a_3 + \frac{b_3}{E_{truth}}$$

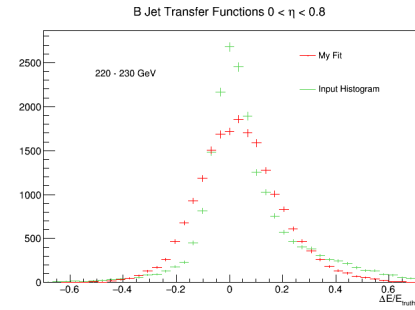
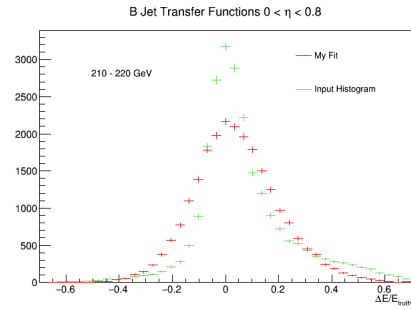
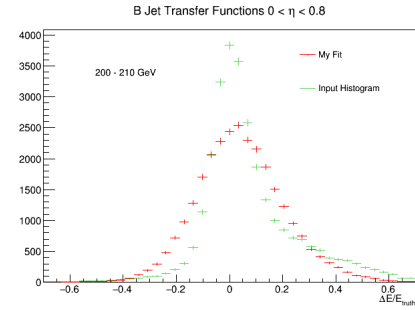
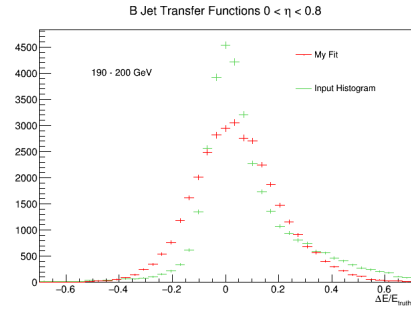
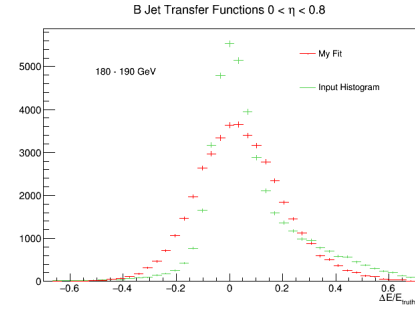
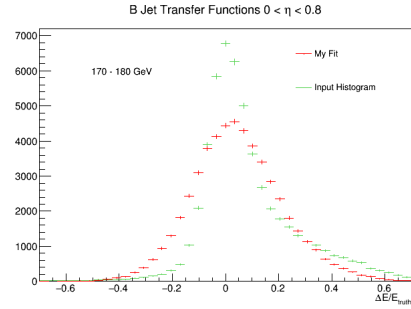
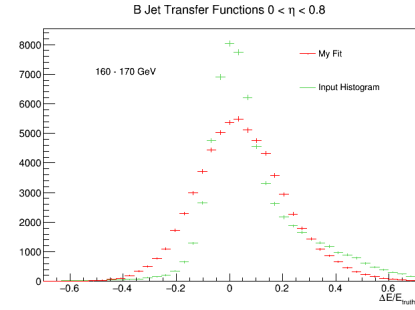
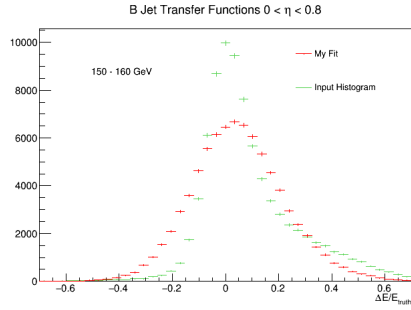
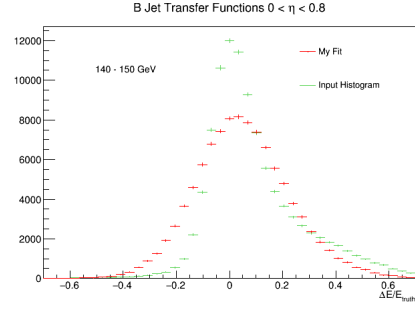
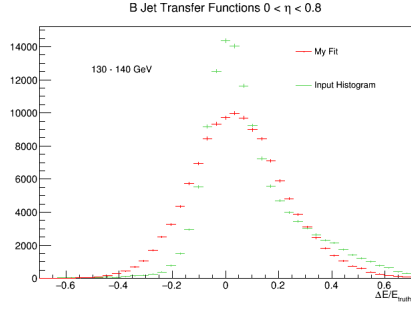
$$\mu_2 = a_4 + \frac{b_4}{\sqrt{E_{truth}}}$$

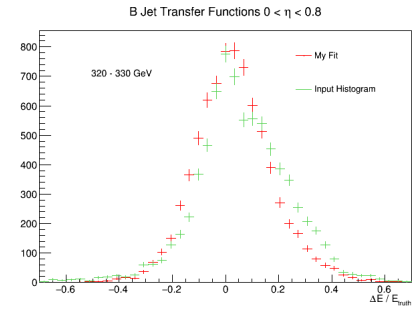
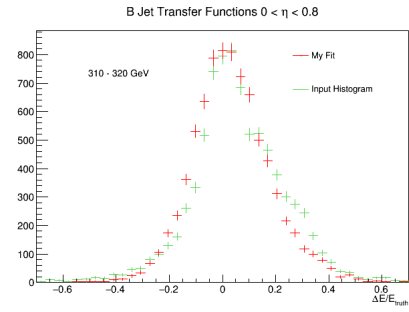
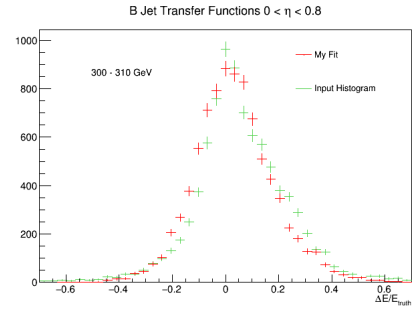
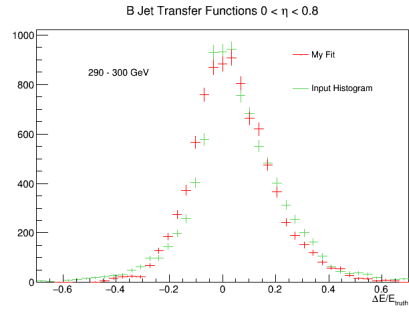
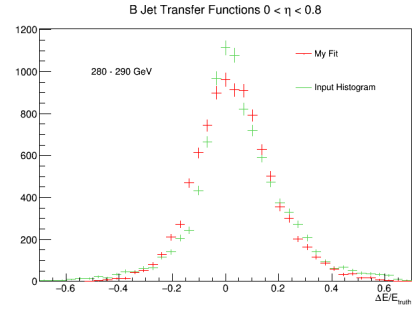
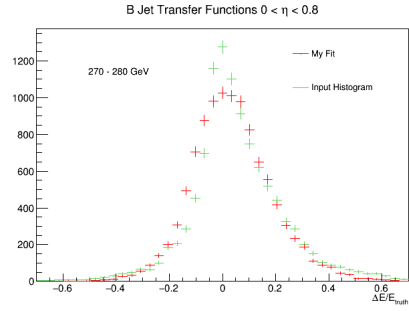
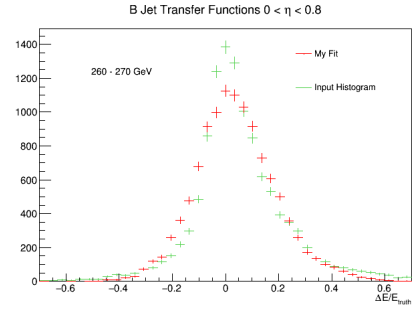
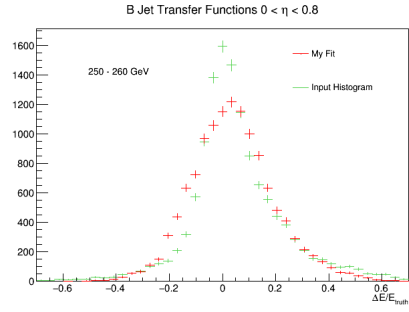
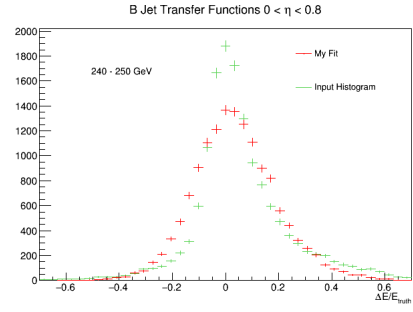
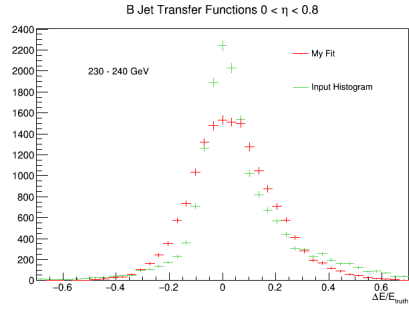
$$\sigma_2 = a_5 + b_5 E_{truth}$$

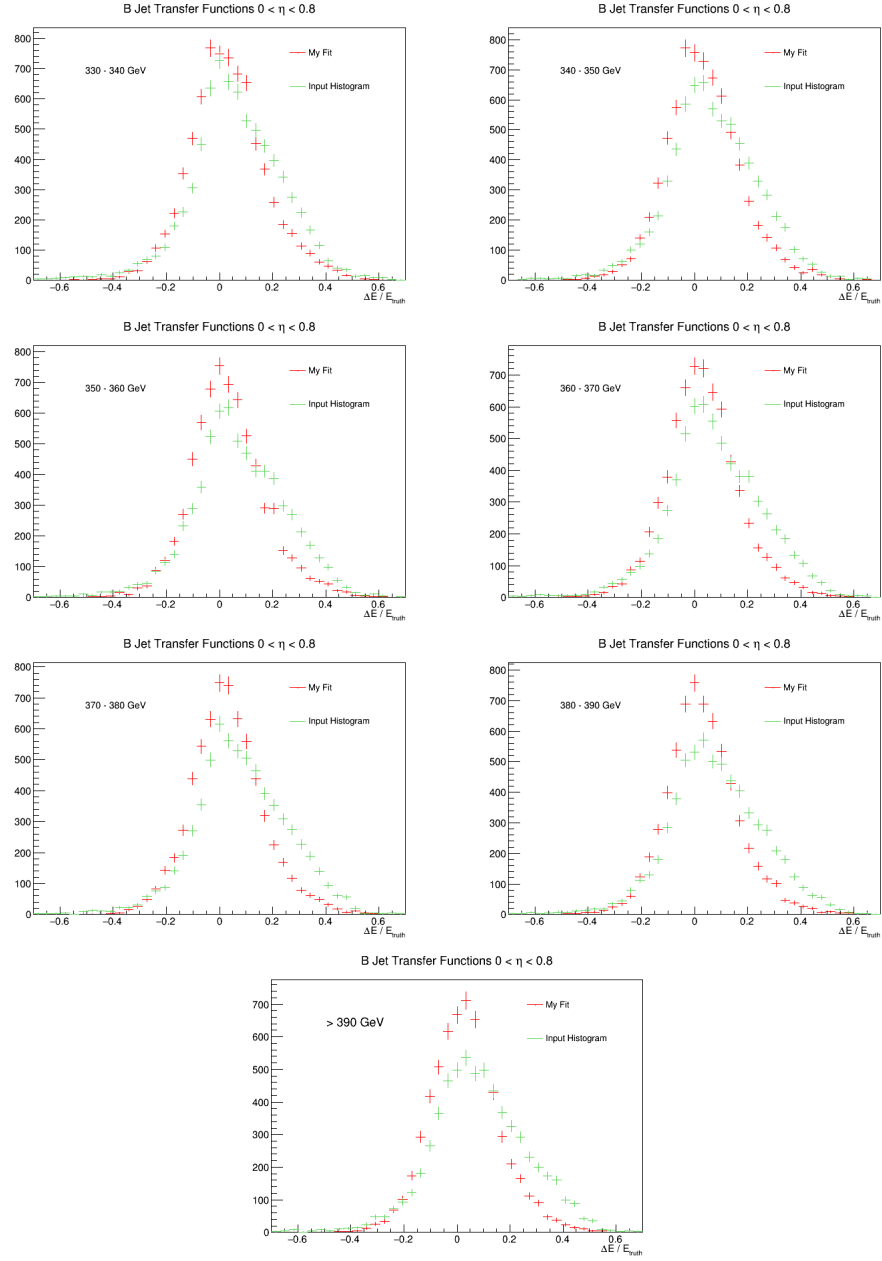
Eta Region 1

Transfer Function Parameters	
a_1	-0.00432154
b_1	1.8055
a_2	0.0240412
b_2	1.40853
a_3	0.478436
b_3	7.89636
a_4	-0.00017881
b_4	1.20603
a_5	0.231936
b_5	-0.000168297



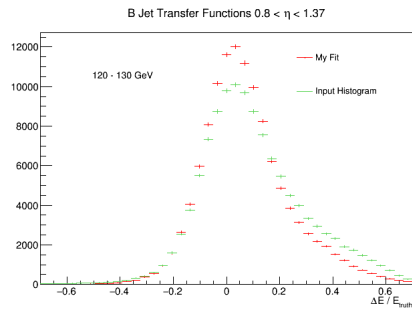
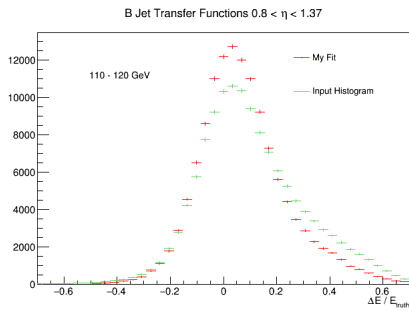
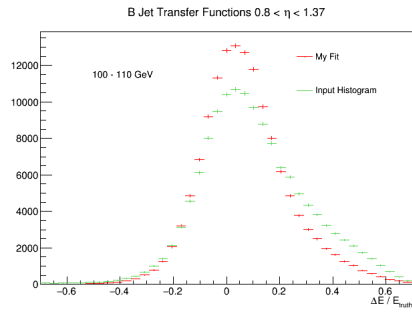
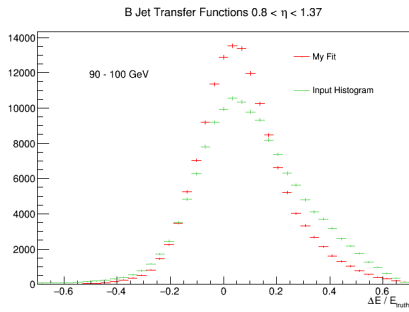
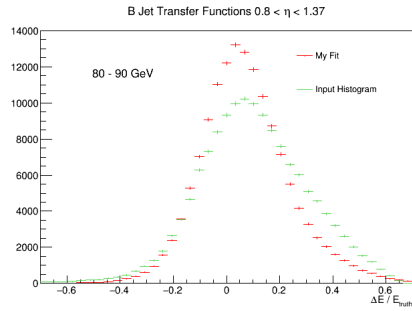
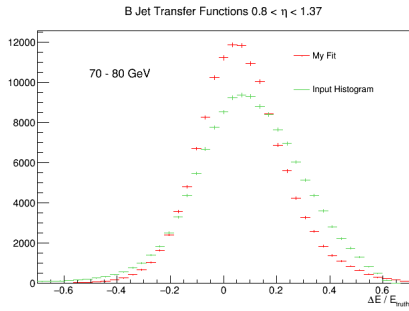
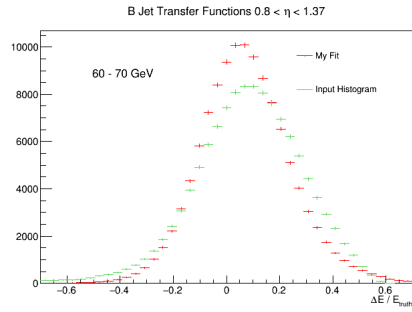
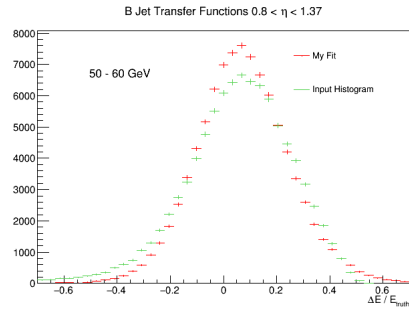
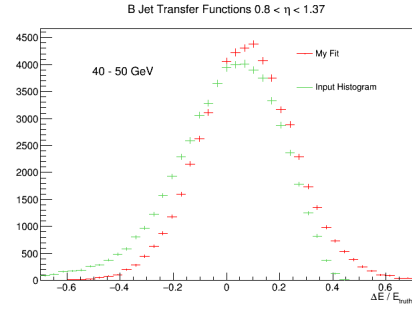
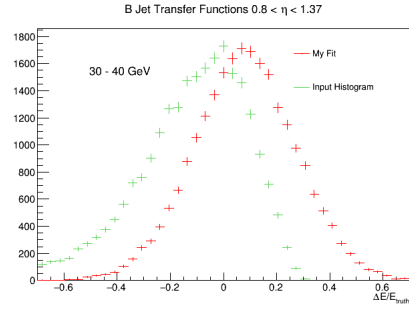


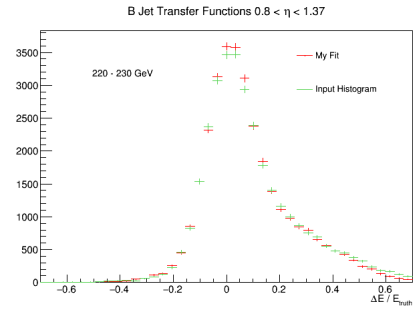
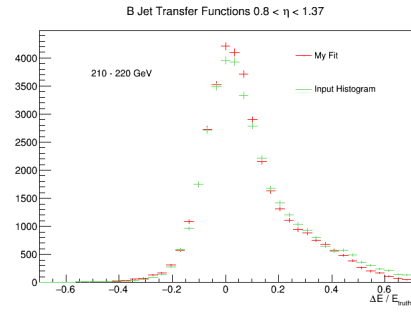
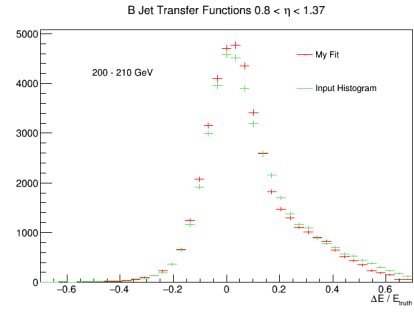
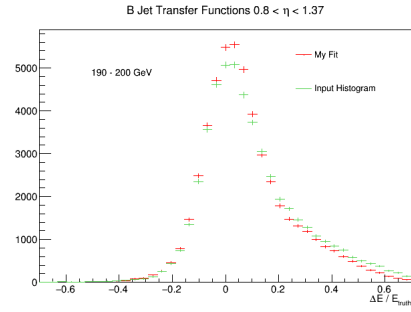
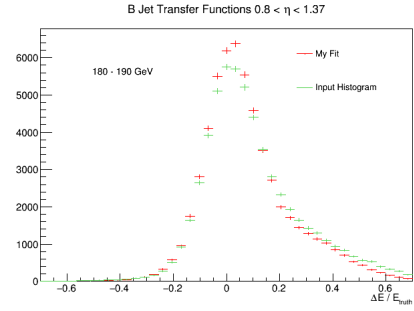
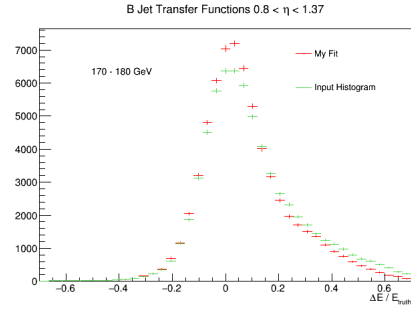
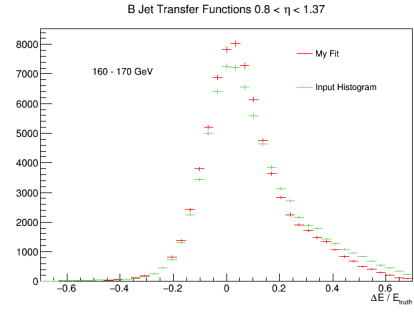
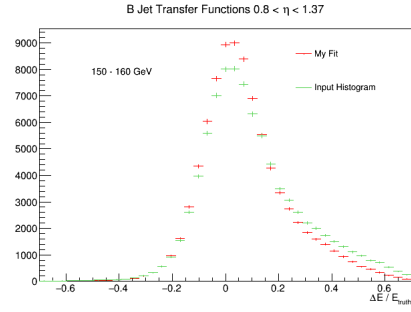
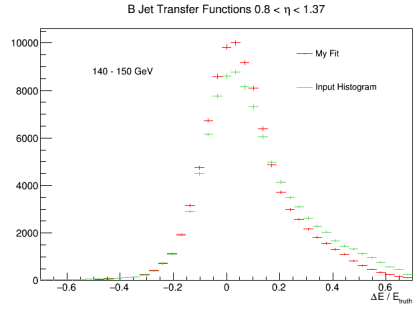
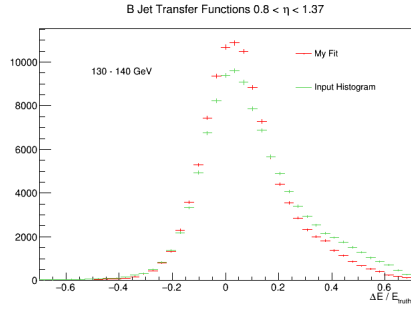


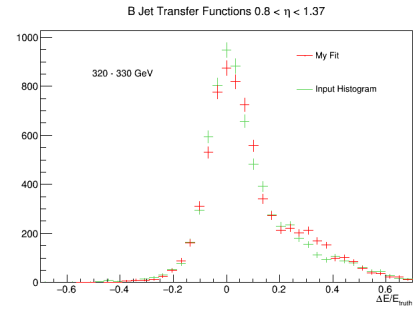
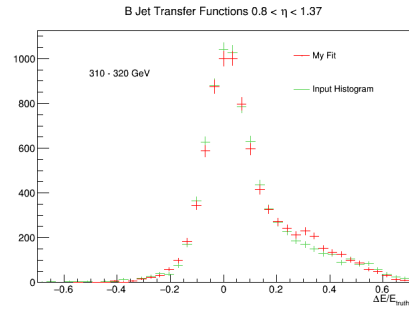
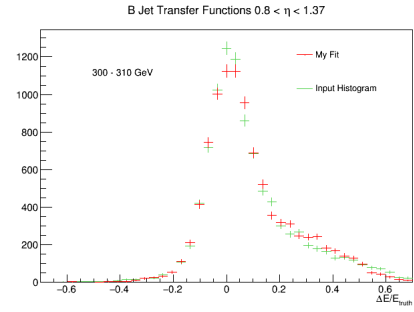
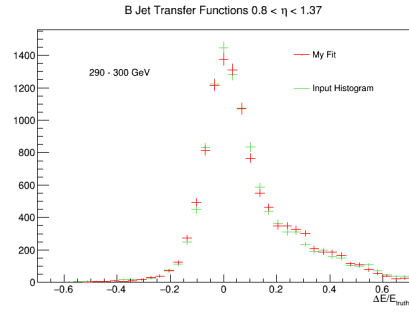
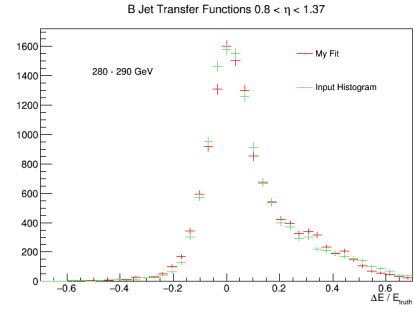
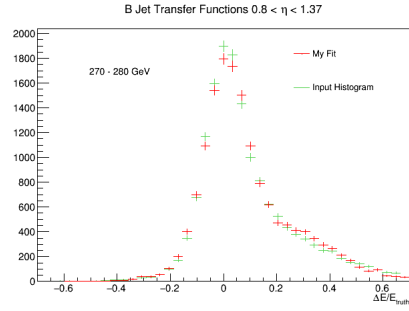
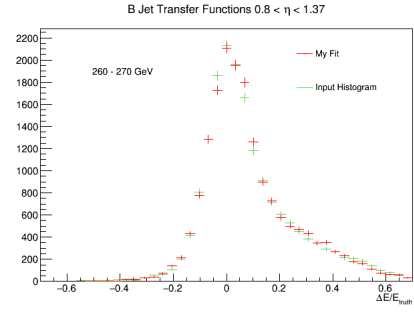
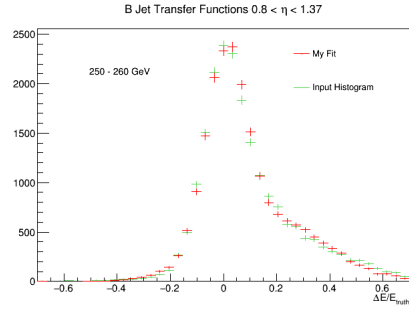
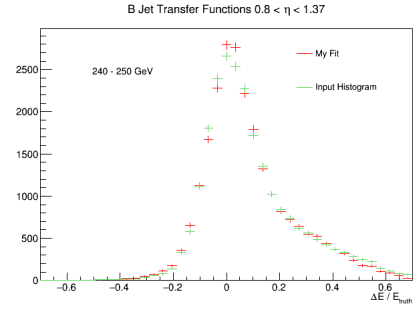
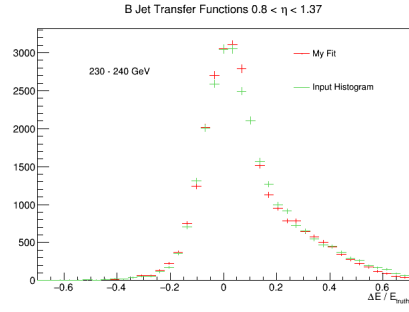


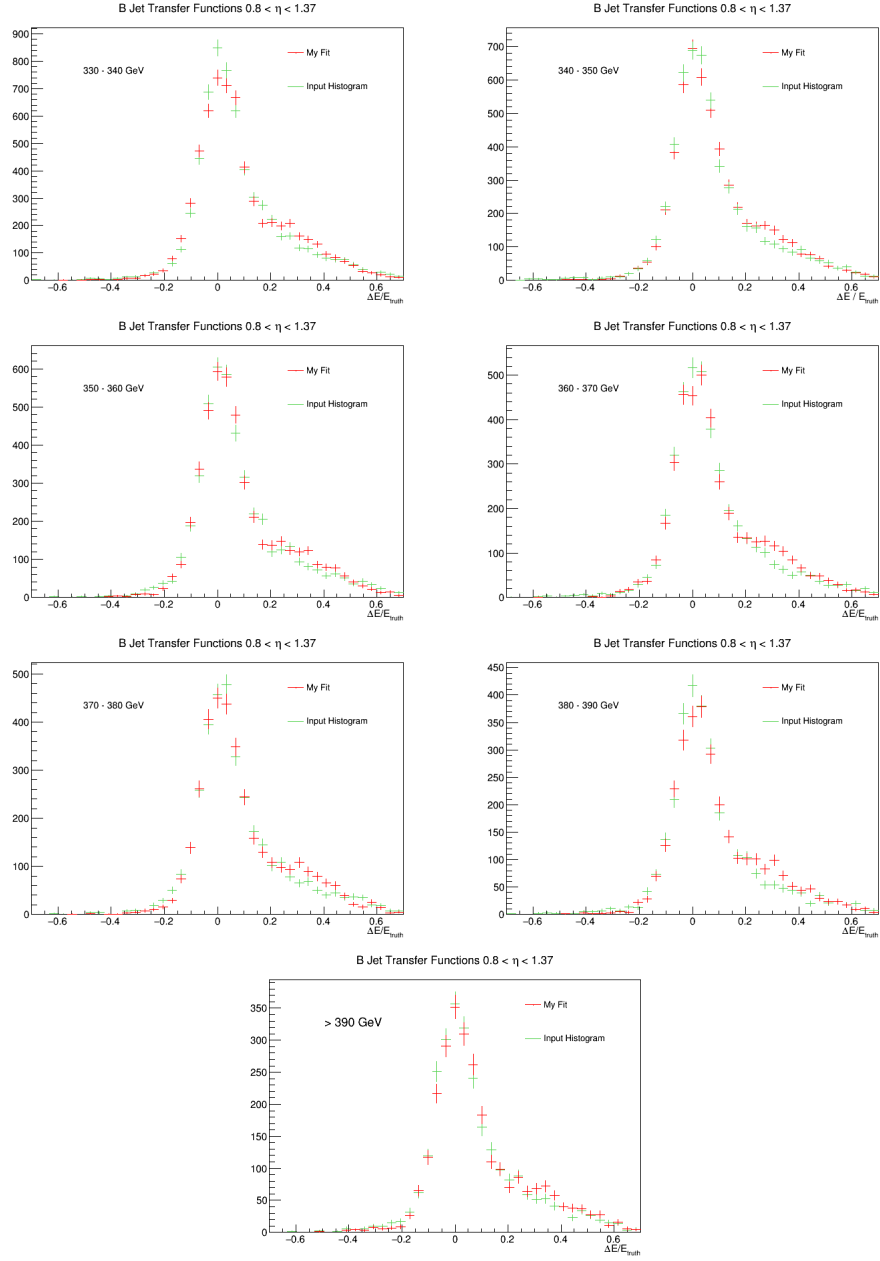
Eta Region 2

Transfer Function Parameters	
a_1	-0.00379513
b_1	3.30386
a_2	0.0180834
b_2	0.943906
a_3	0.27372
b_3	7.47193
a_4	0.2495
b_4	-1.09052
a_5	0.216925
b_5	$-4.13625 \cdot 10^{-5}$



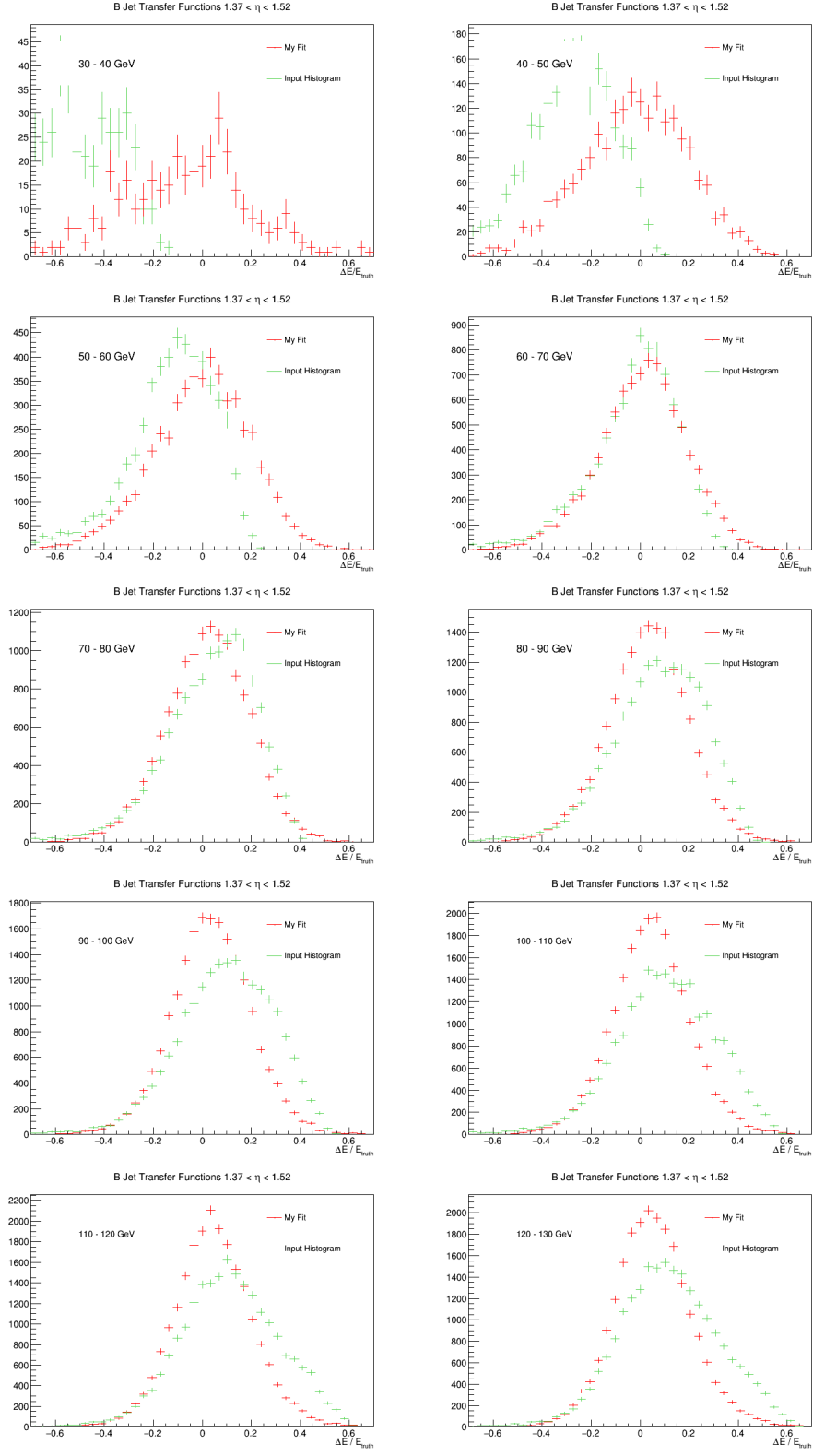


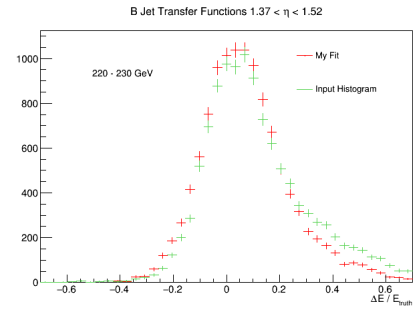
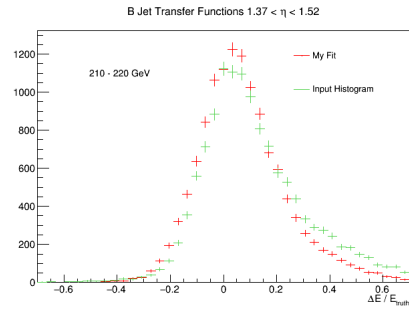
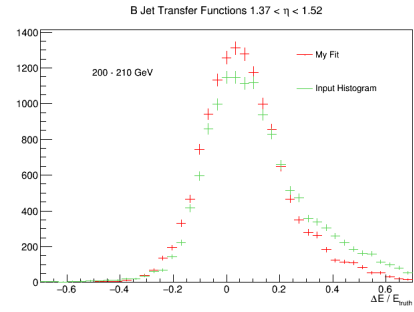
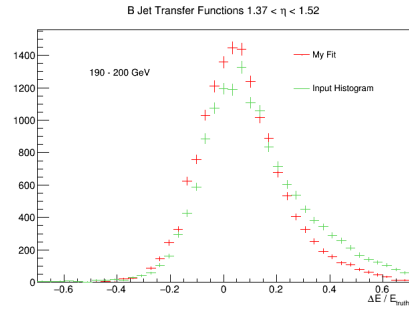
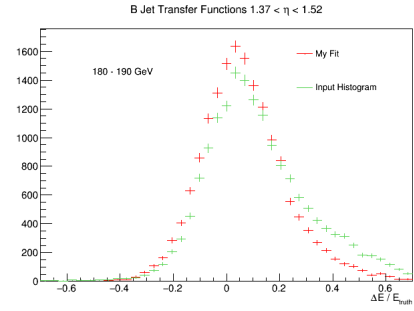
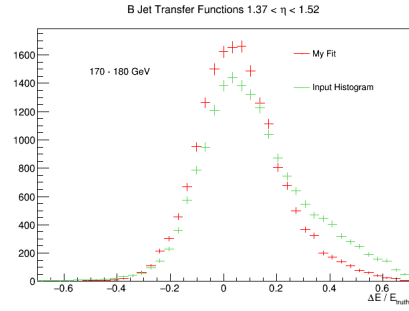
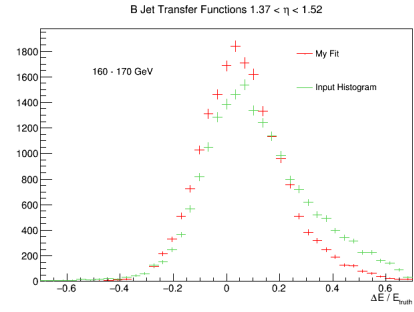
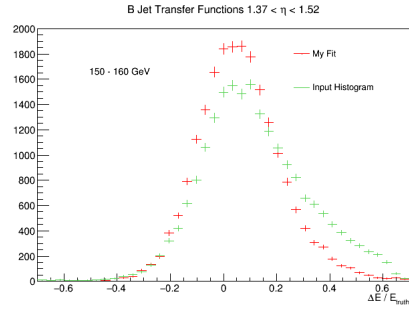
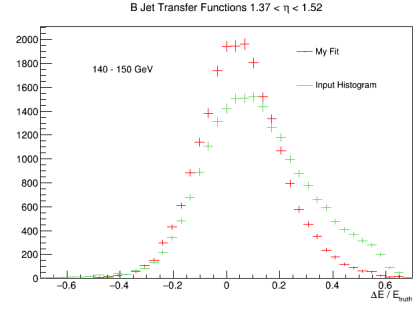
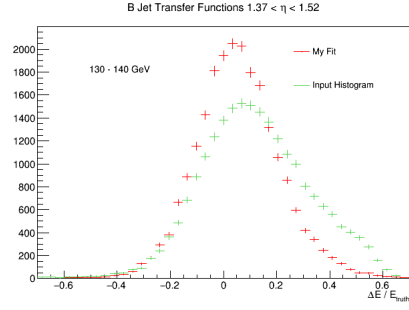


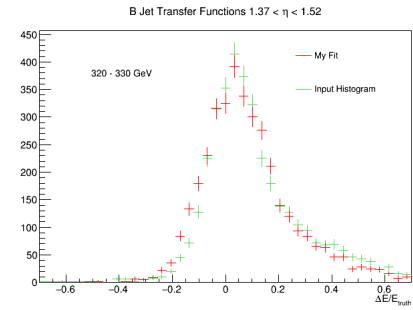
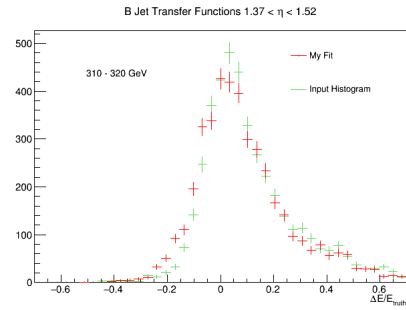
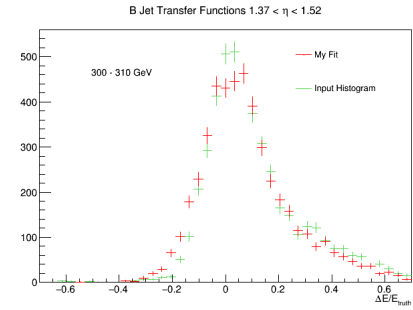
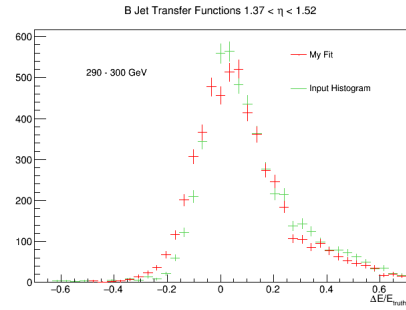
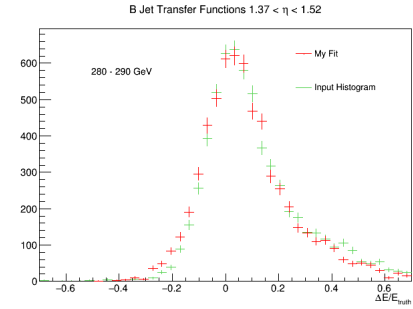
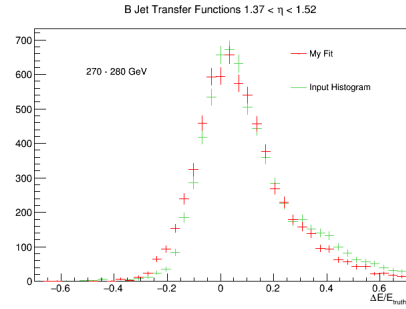
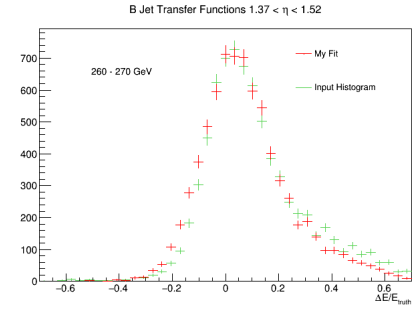
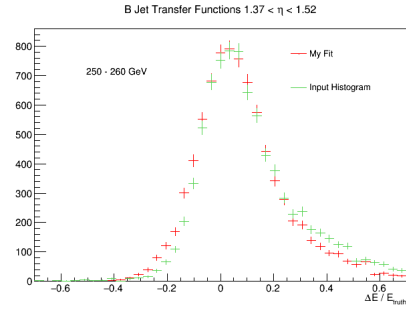
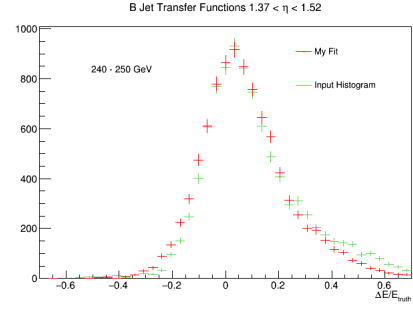
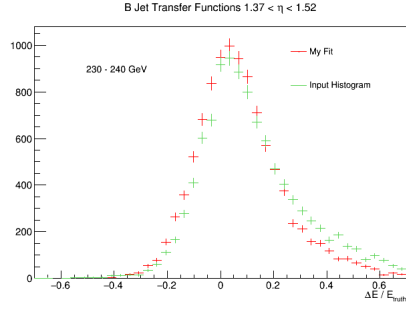


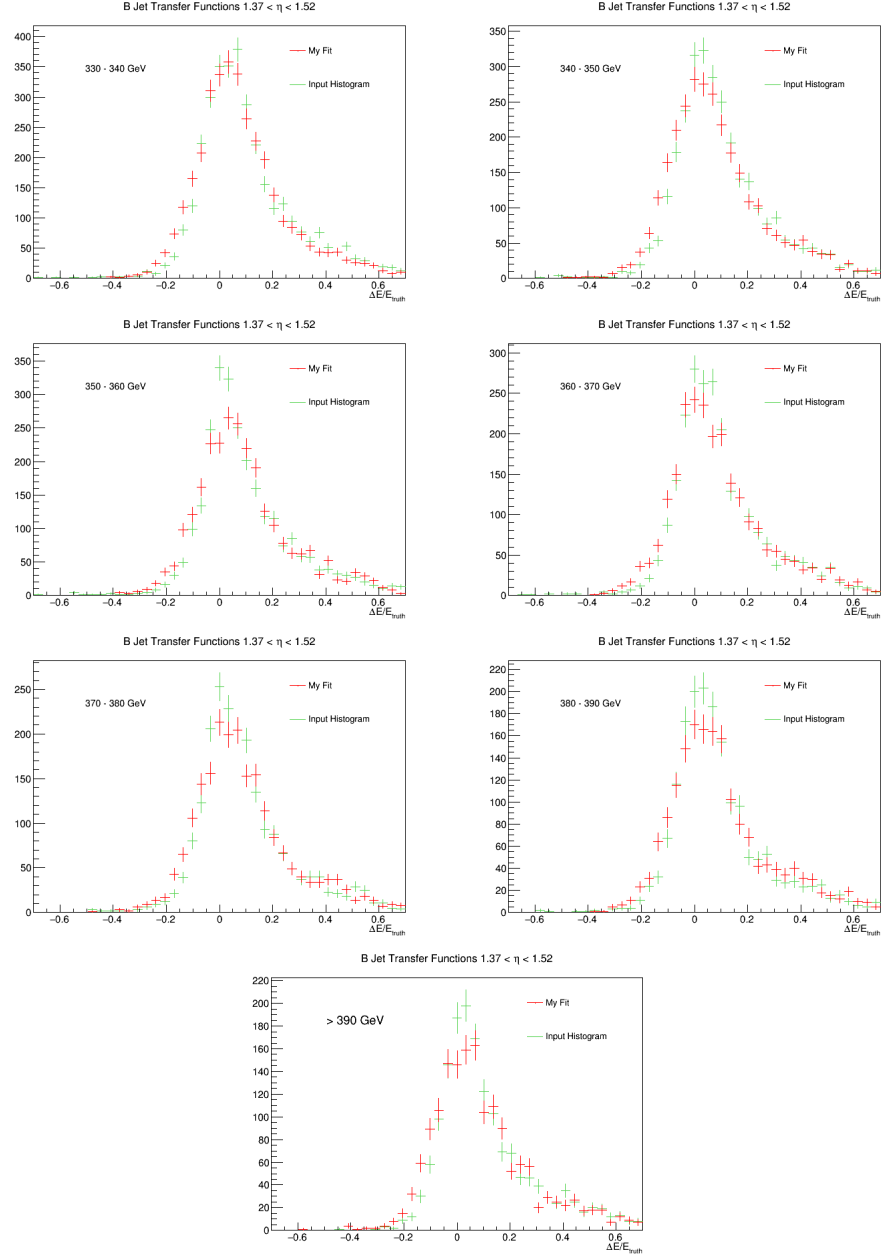
Eta Region 3

Transfer Function Parameters	
a_1	0.0148761
b_1	2.62929
a_2	0.0733037
b_2	0.634043
a_3	0.171434
b_3	18.4382
a_4	0.475686
b_4	-4.38242
a_5	0.190552
b_5	$9.55331 \cdot 10^{-5}$



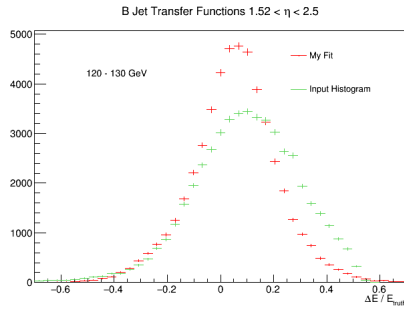
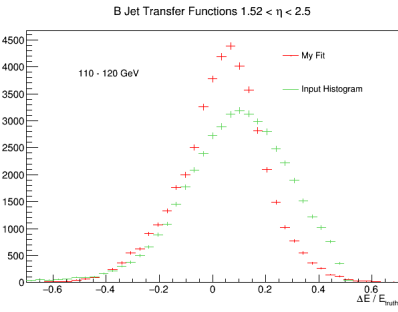
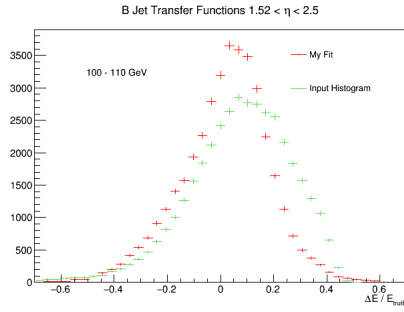
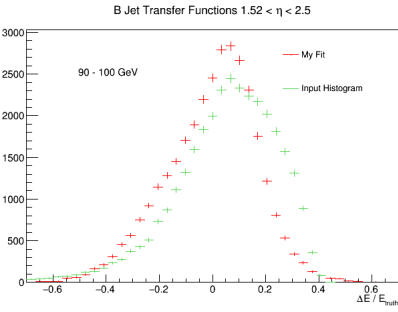
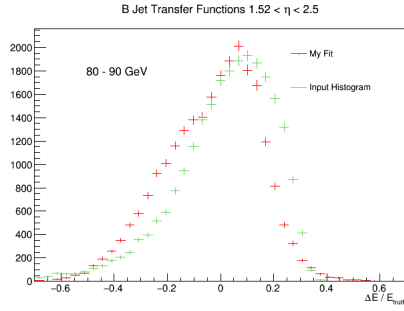
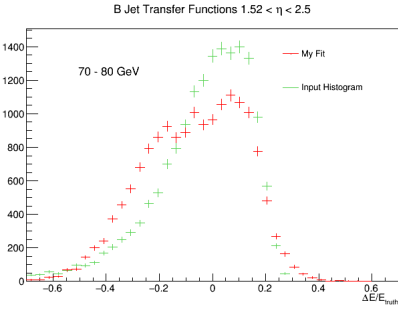
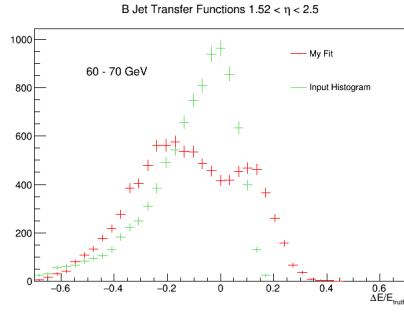
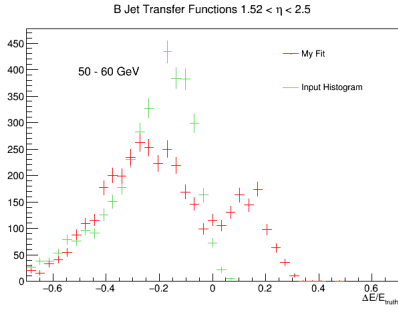
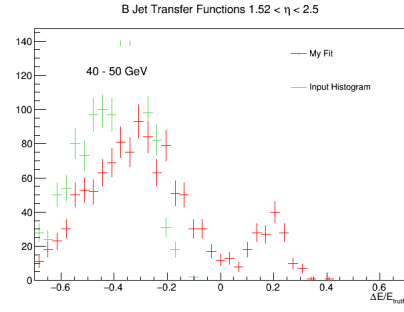
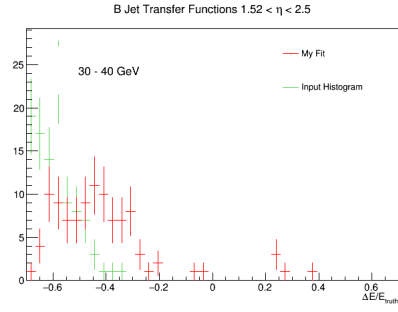


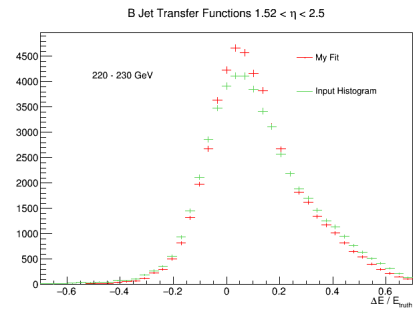
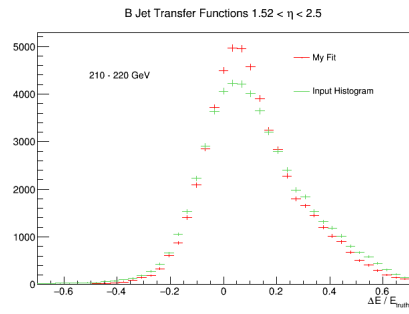
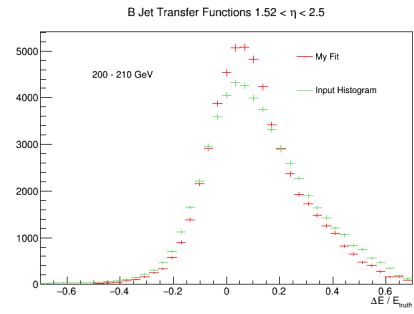
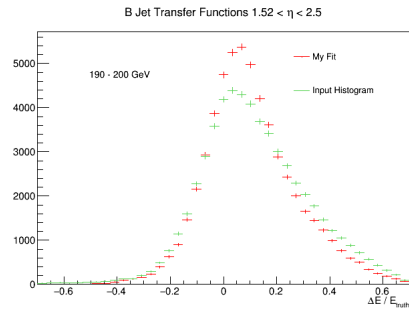
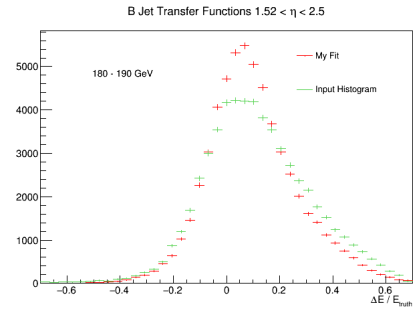
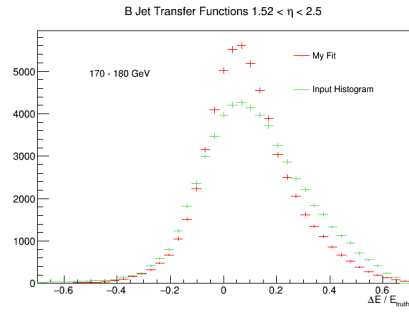
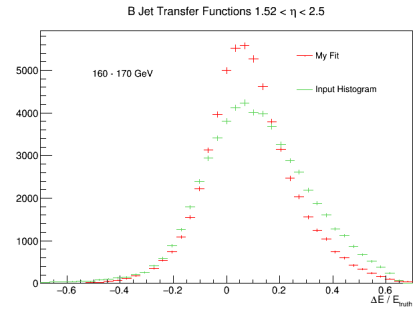
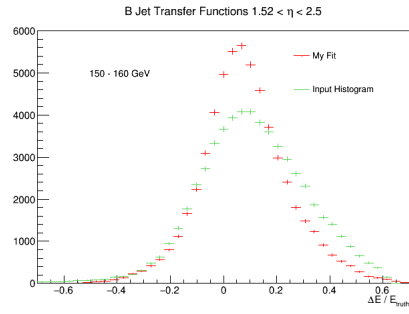
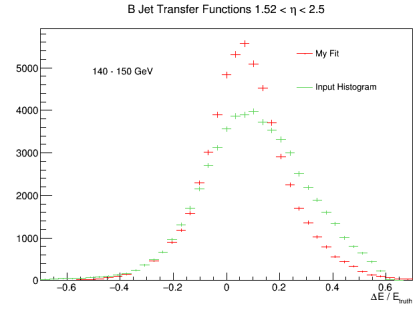
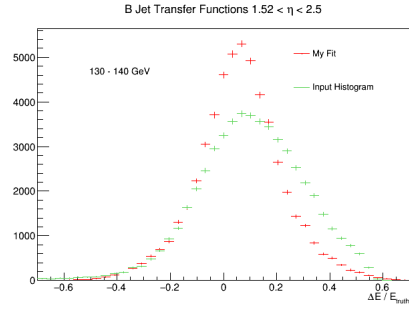


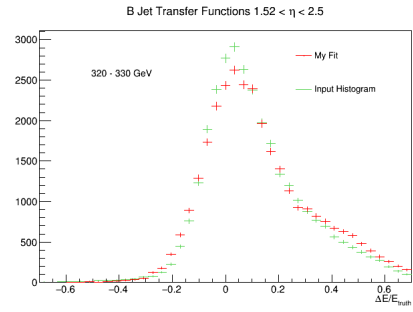
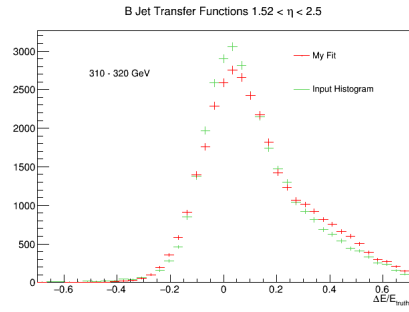
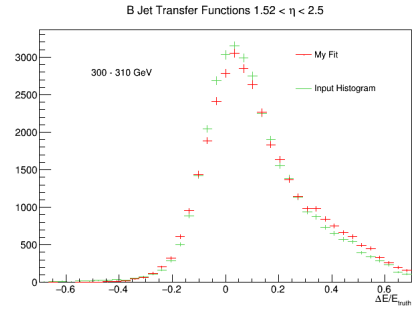
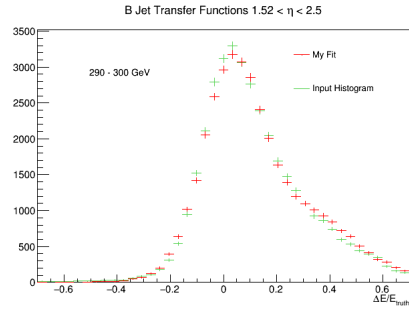
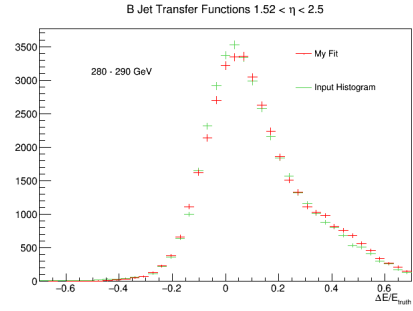
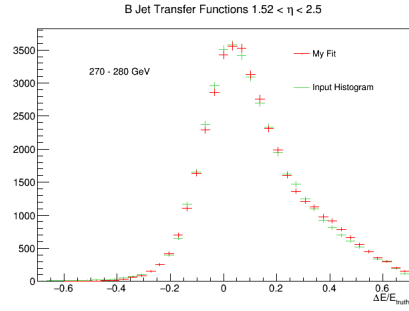
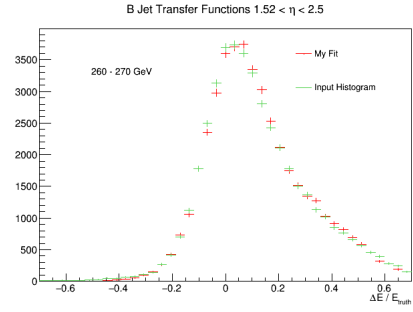
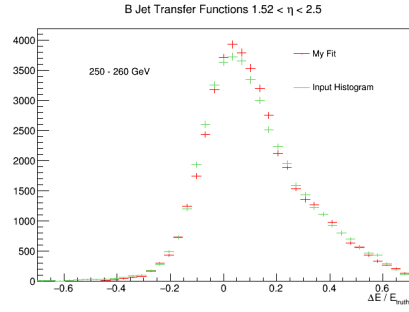
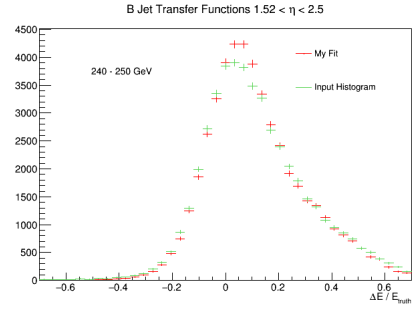
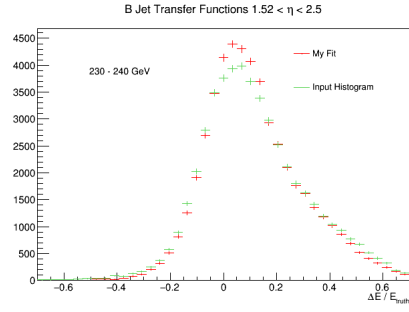


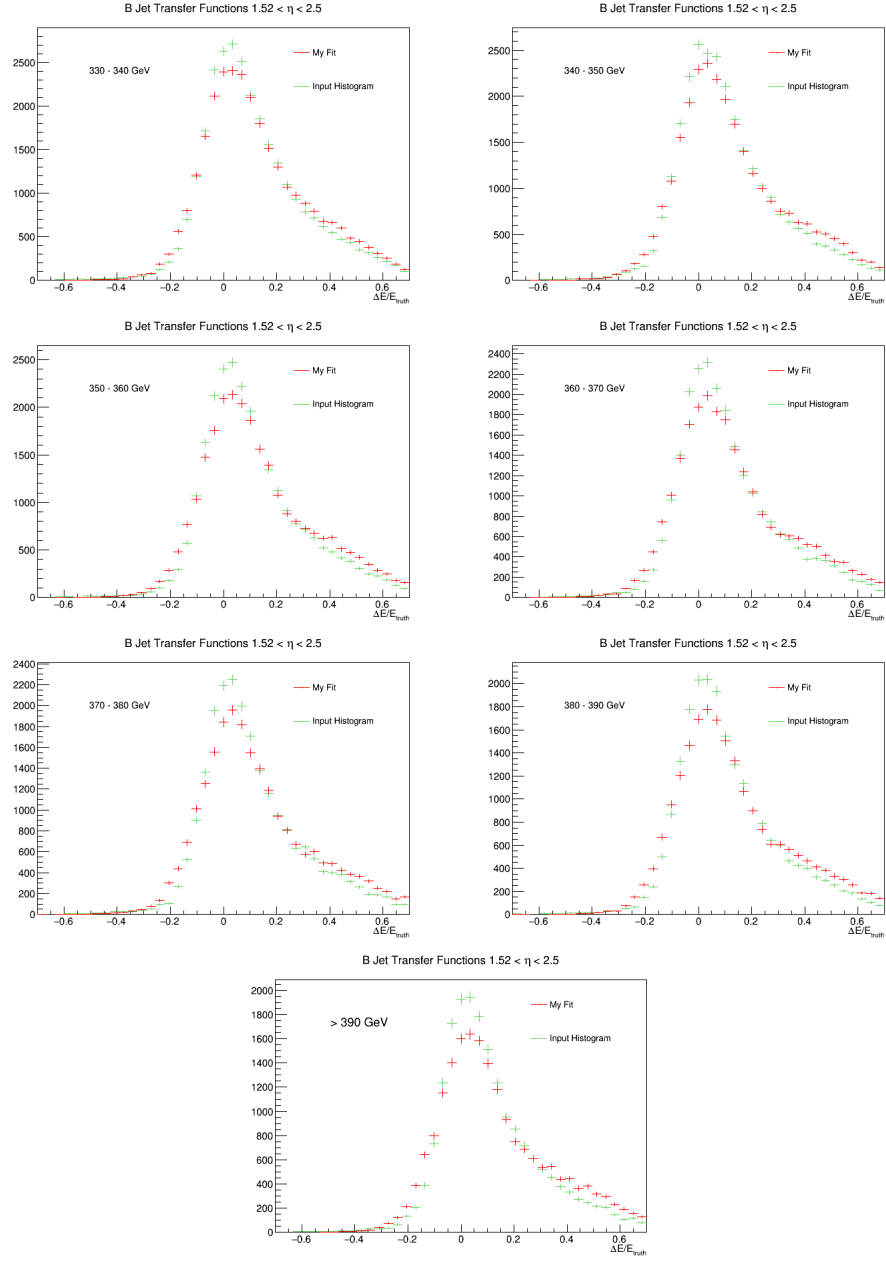
Eta Region 4

Transfer Function Parameters	
a_1	0.000708911
b_1	8.22719
a_2	0.138447
b_2	-0.544192
a_3	0.111233
b_3	95.2936
a_4	0.608303
b_4	-6.32026
a_5	0.166763
b_5	0.000197378









6.3 Electron Transfer Functions

The functional form used for the electron transfer functions was

$$\frac{1}{\sqrt{2\pi}(\sigma_1 + p_3\sigma_2)} \left(e^{-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2} + p_3 e^{-\frac{1}{2}\left(\frac{x-\mu_2}{\sigma_2}\right)^2} \right)$$

where $x = \frac{E_{truth}-E_{reco}}{E_{truth}}$. Note that all energies are measured in units of GeV. The parametrizations used for the double Gaussian parameters are

$$\mu_1 = a_1 + b_1 E_{truth}$$

$$\sigma_1 = a_2 + \frac{b_2}{\sqrt{E_{truth}}}$$

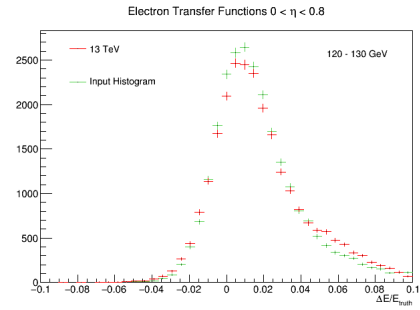
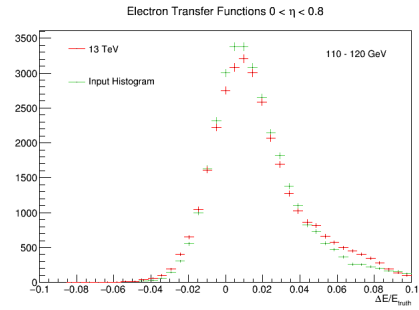
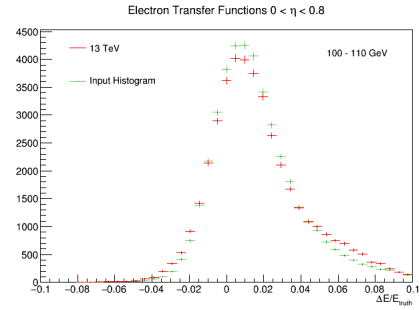
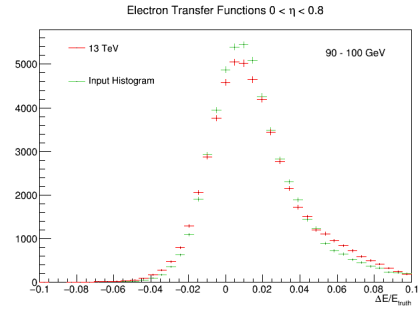
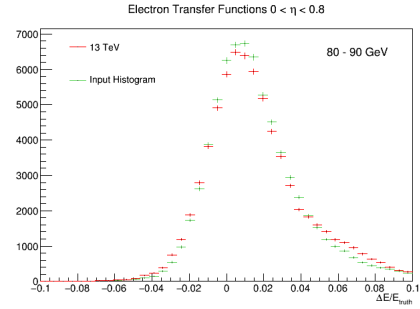
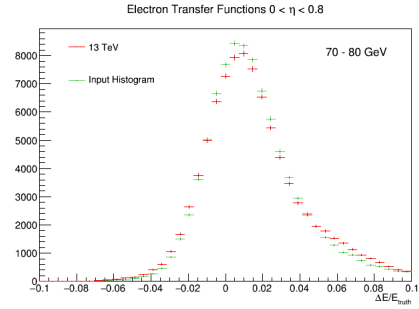
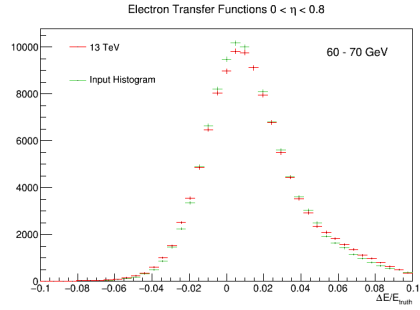
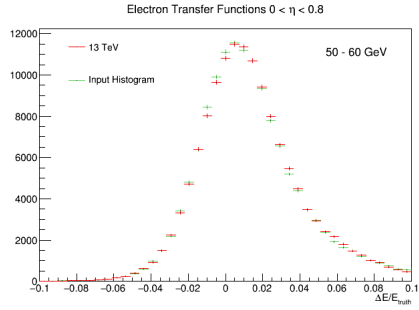
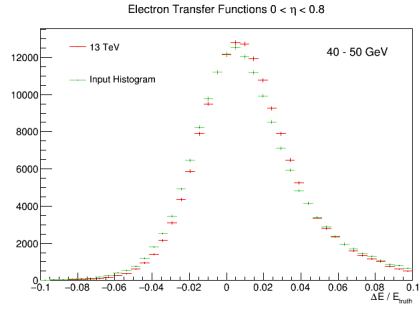
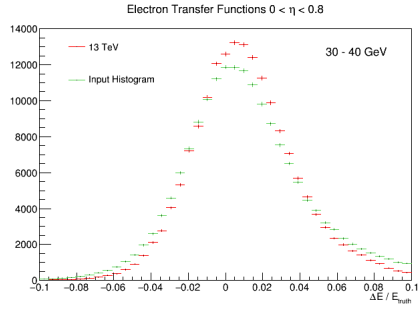
$$p_3 = a_3 + b_3 E_{truth}$$

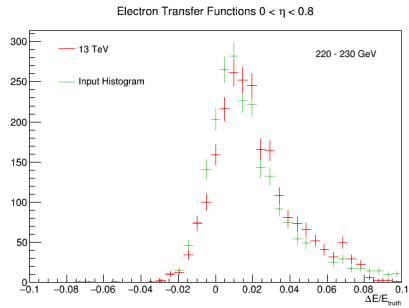
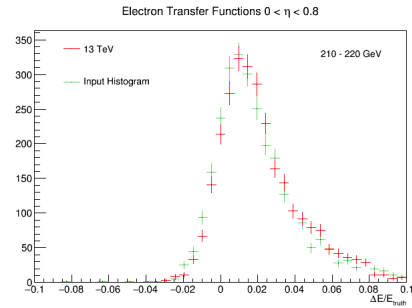
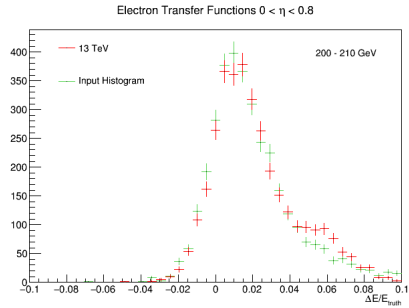
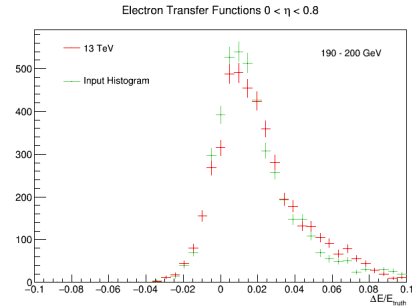
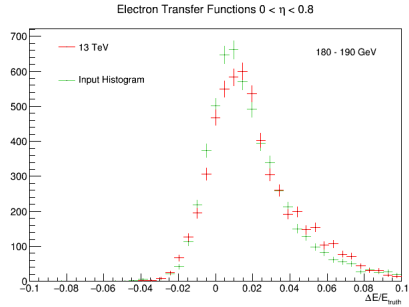
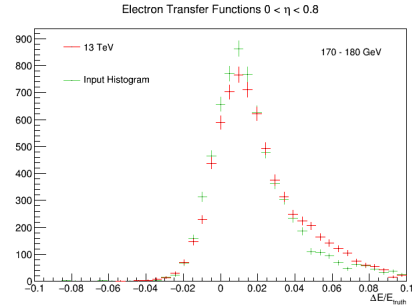
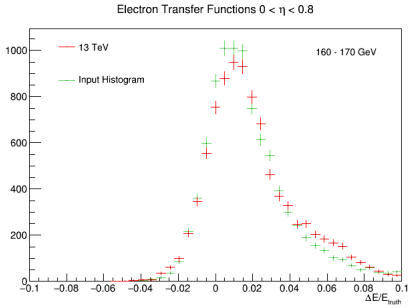
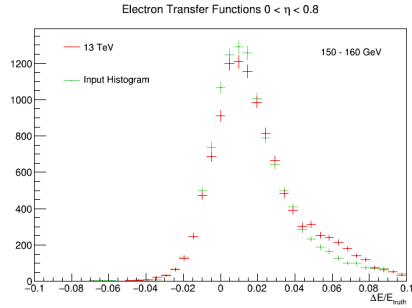
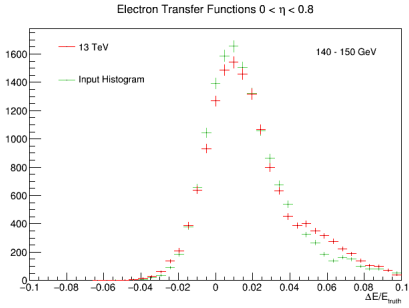
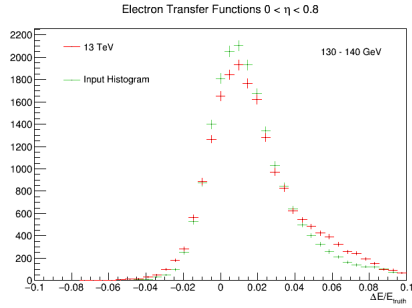
$$\mu_2 = a_4 + \frac{b_4}{\sqrt{E_{truth}}}$$

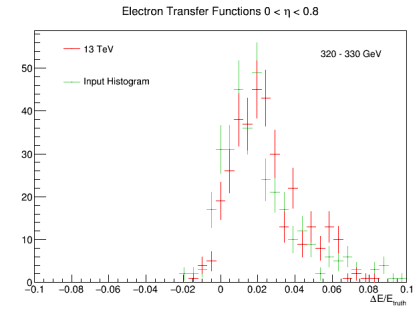
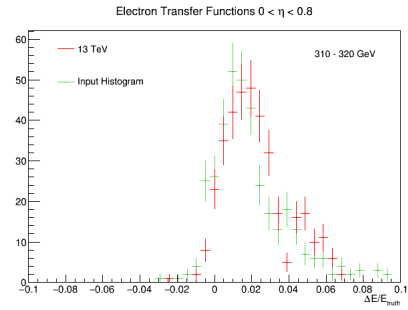
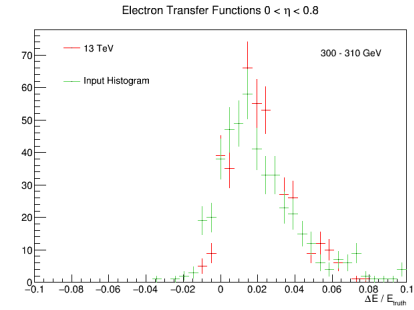
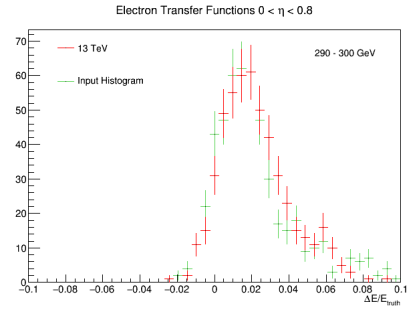
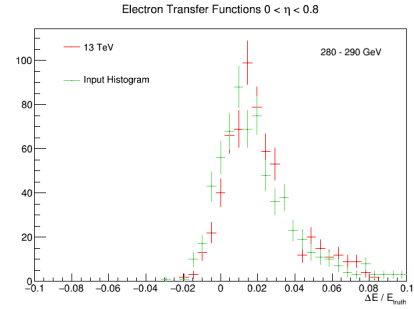
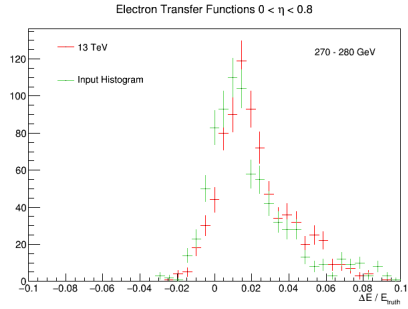
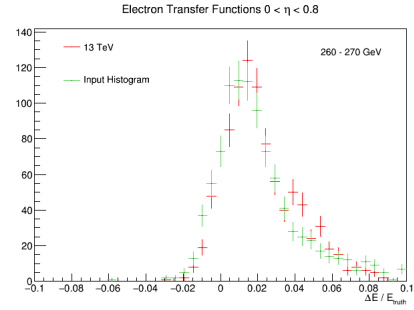
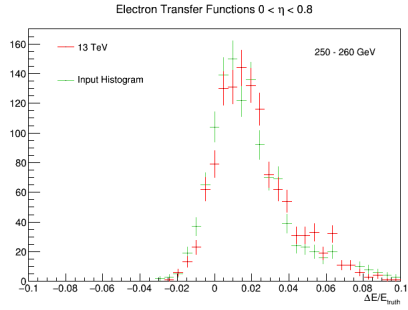
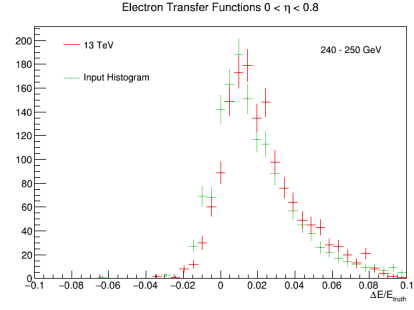
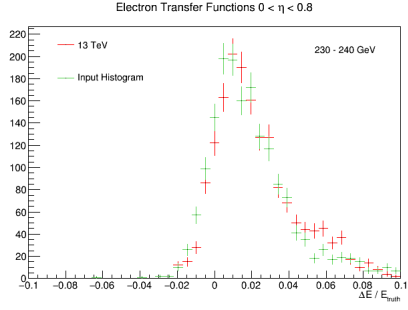
$$\sigma_2 = a_5 + b_5 E_{truth}$$

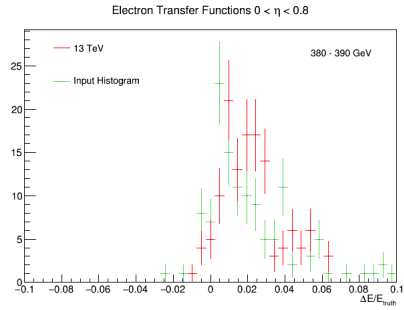
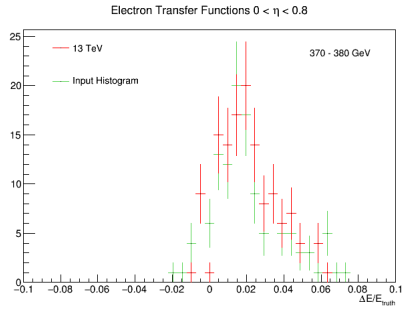
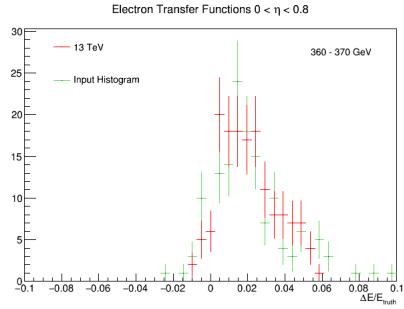
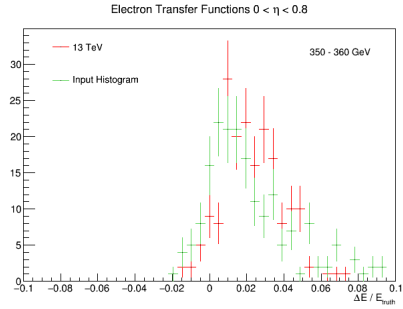
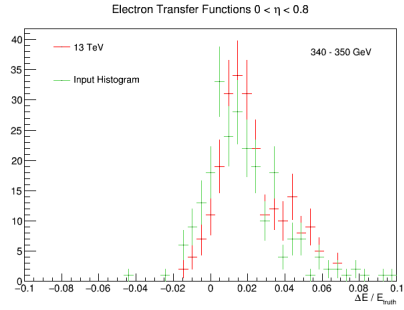
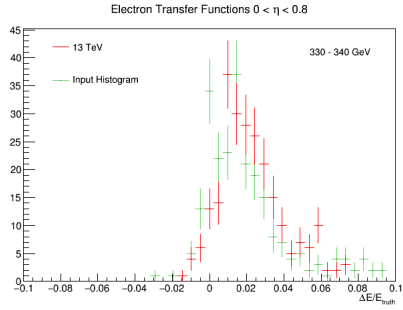
Eta Region 1

Transfer Function Parameters	
a_1	0.00360271
b_1	$3.33603 \cdot 10^{-5}$
a_2	0.0062623
b_2	0.0886328
a_3	0.297893
b_3	0.000589548
a_4	0.0498912
b_4	-0.154182
a_5	0.0401085
b_5	$-7.69083 \cdot 10^{-5}$



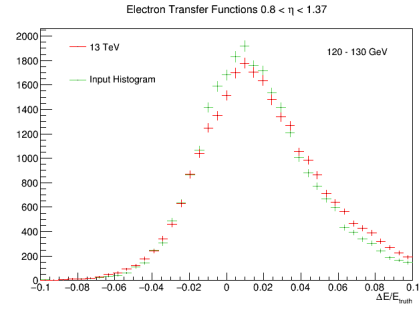
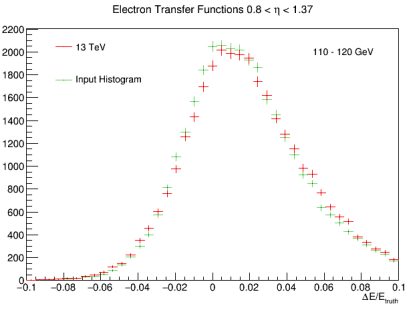
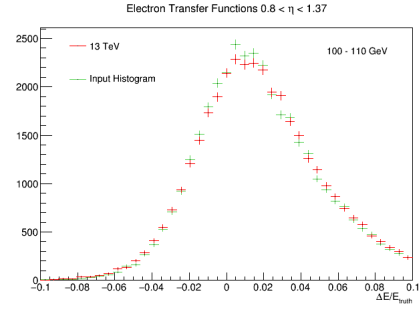
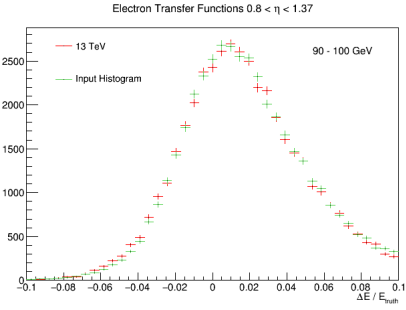
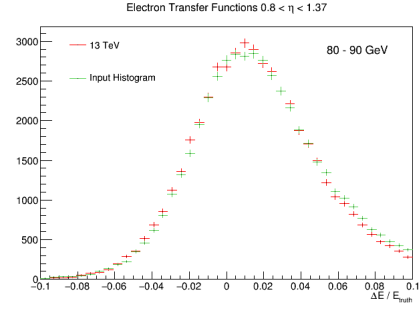
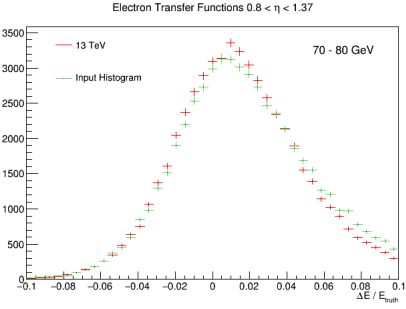
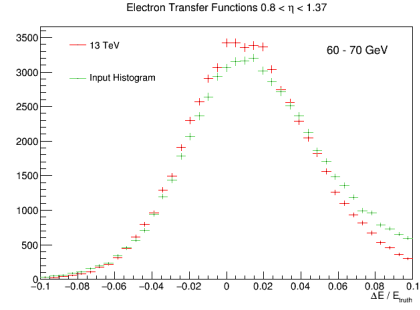
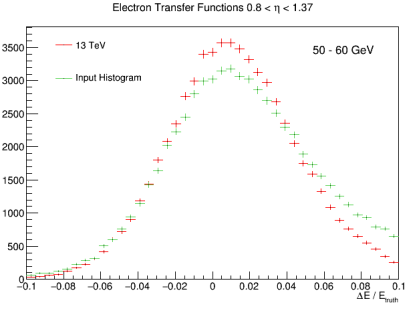
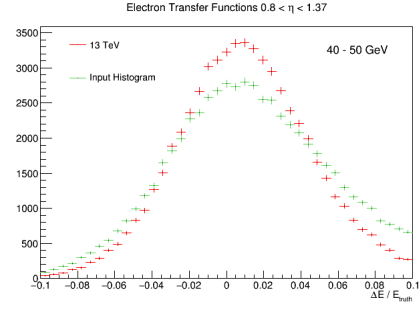
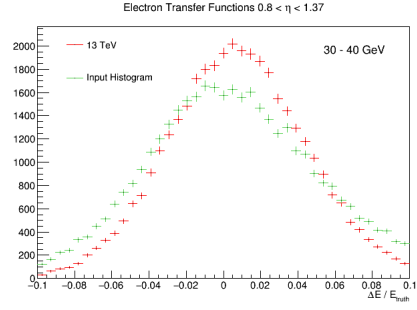


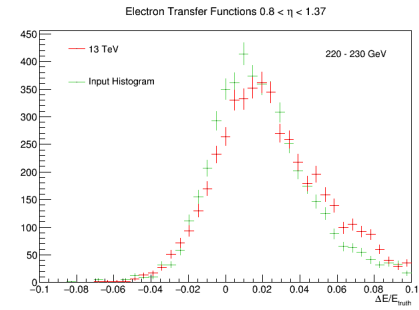
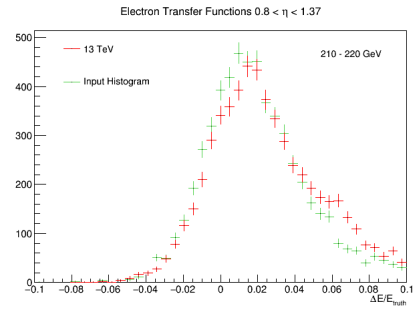
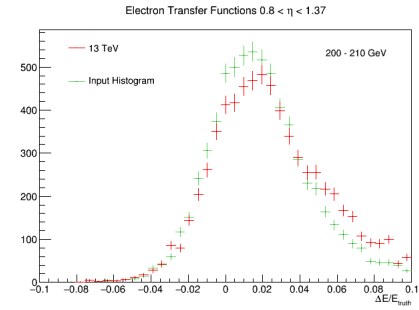
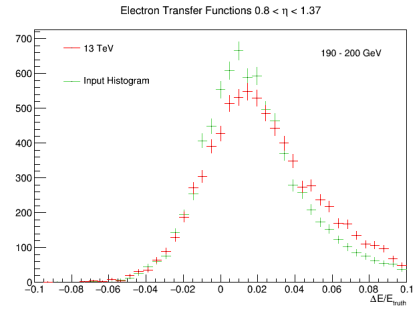
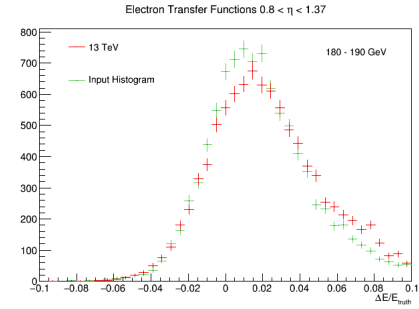
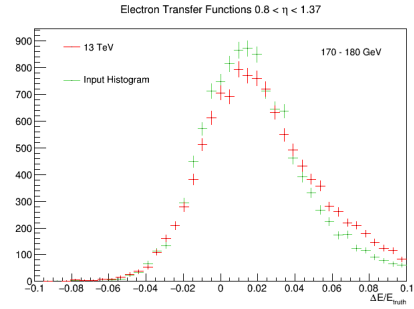
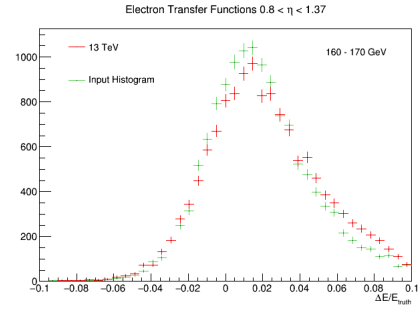
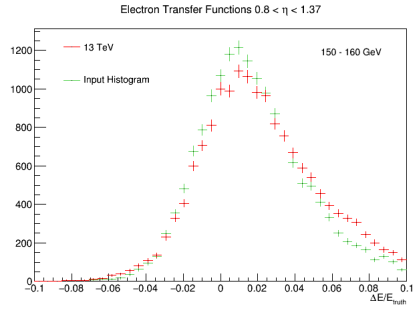
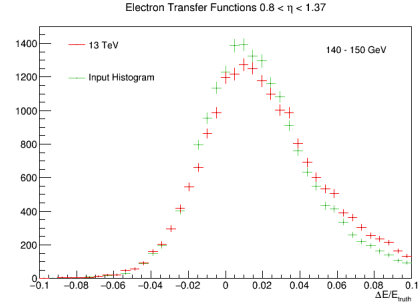
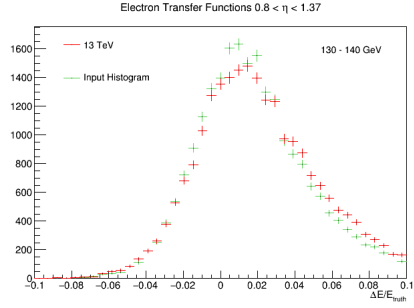


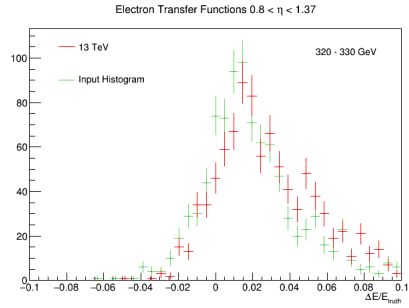
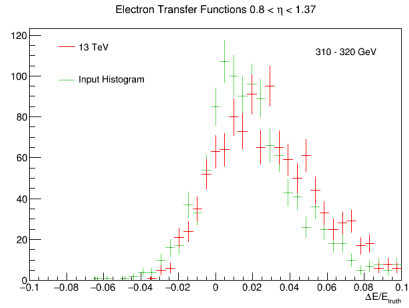
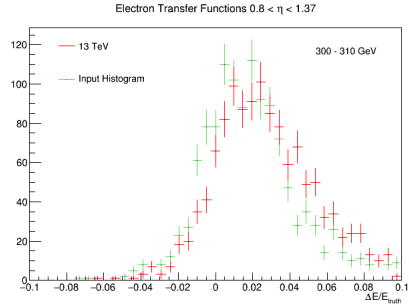
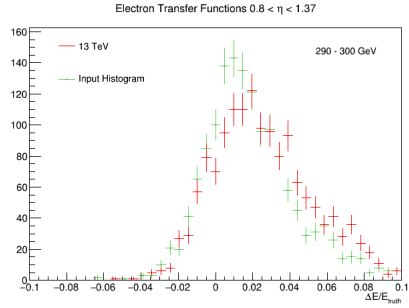
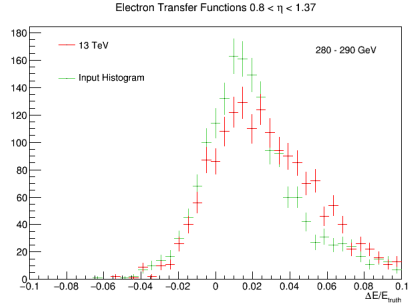
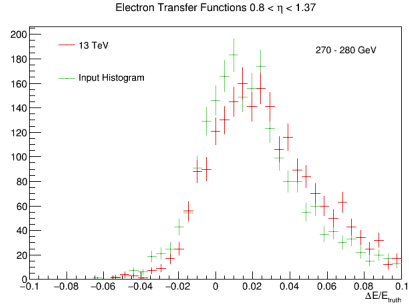
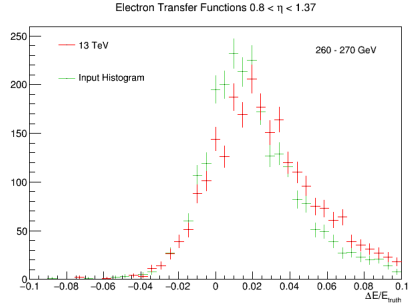
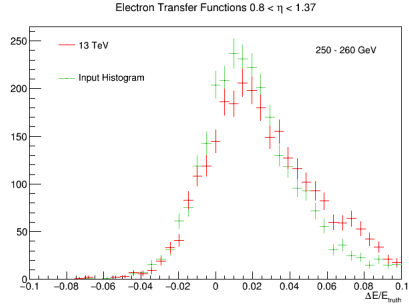
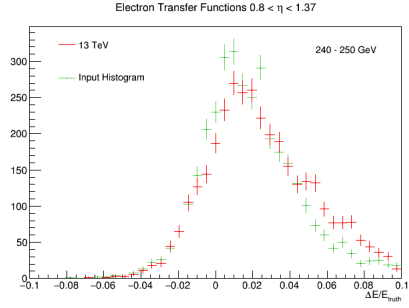
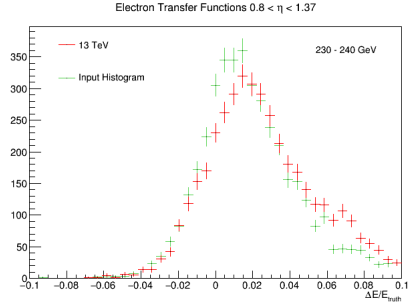


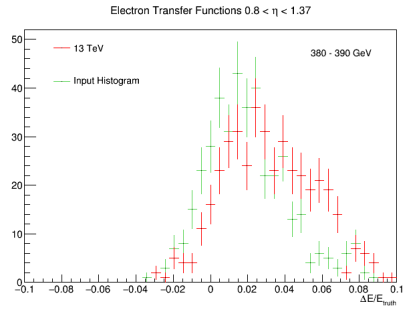
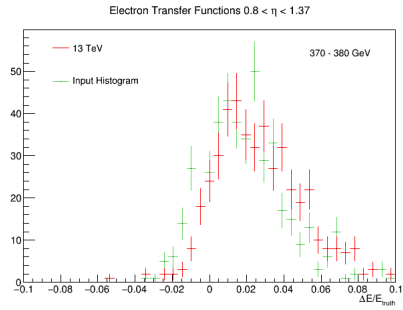
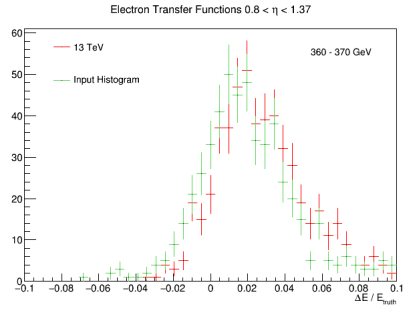
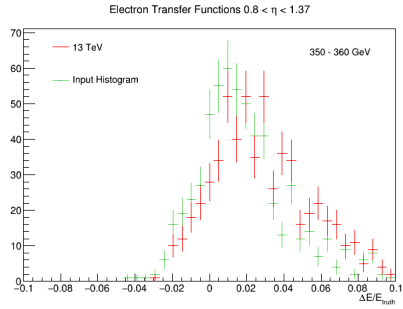
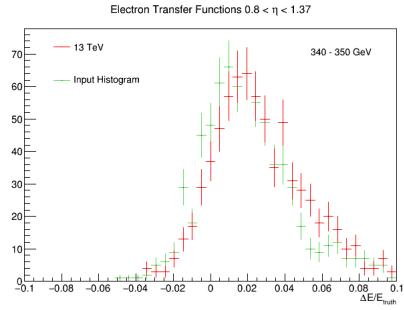
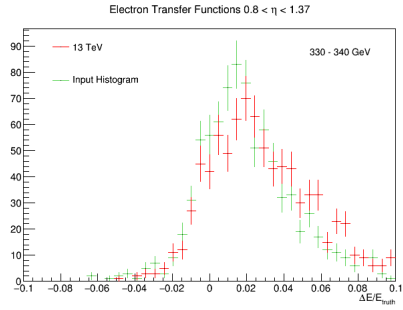
Eta Region 2

Transfer Function Parameters	
a_1	0.00285929
b_1	$3.30998 \cdot 10^{-5}$
a_2	0.0089998
b_2	0.14087
a_3	0.646116
b_3	0.00151672
a_4	0.0562376
b_4	-0.261209
a_5	0.0453541
b_5	$-5.33366 \cdot 10^{-5}$



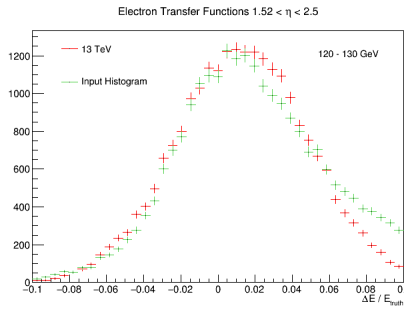
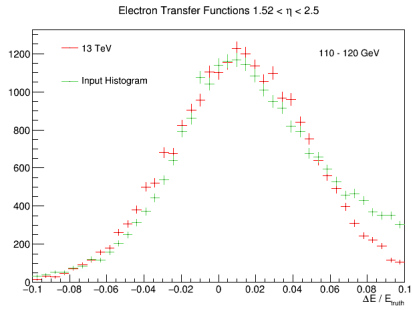
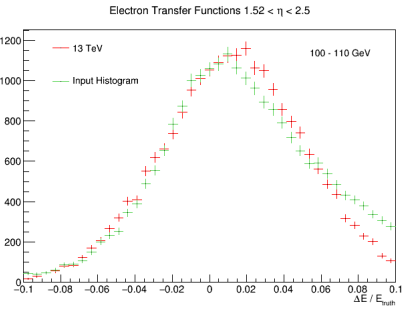
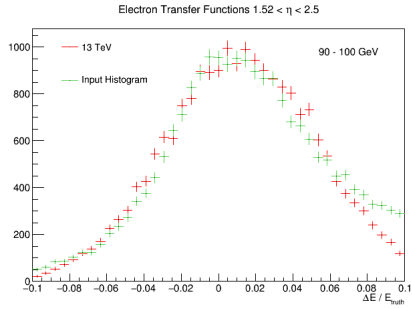
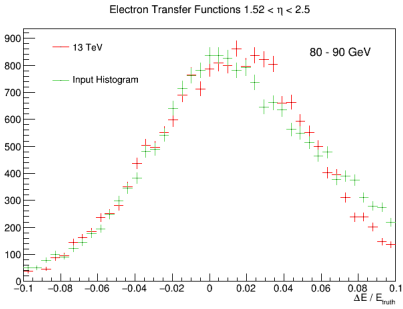
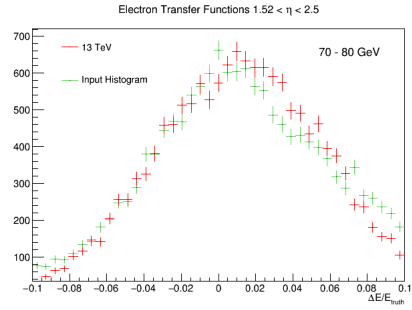
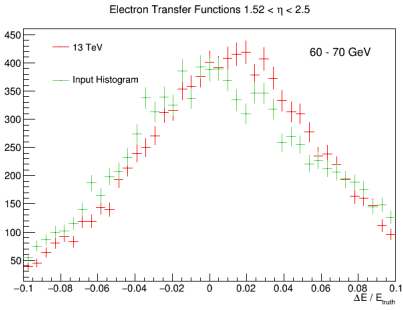
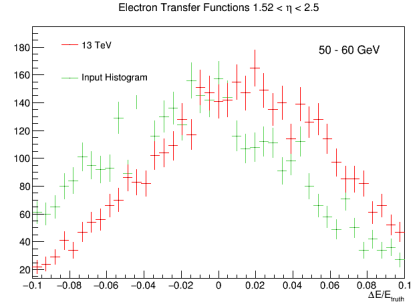
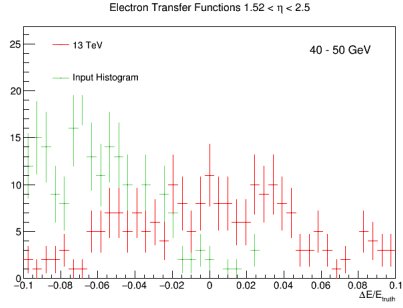


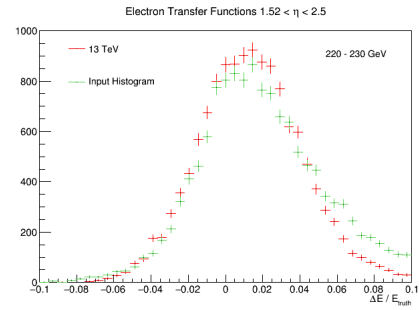
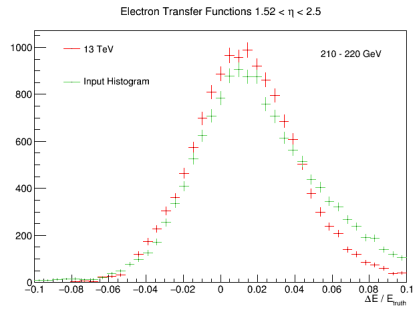
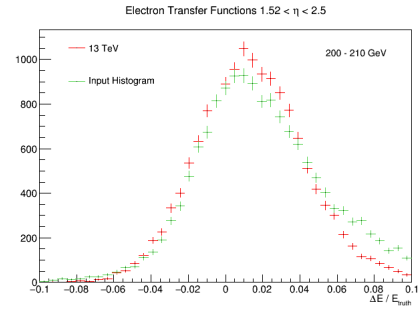
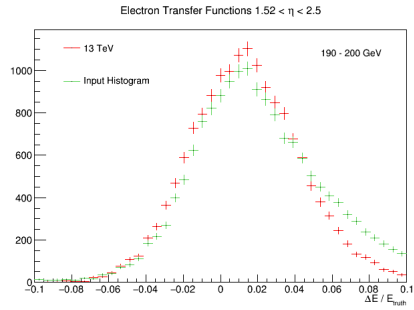
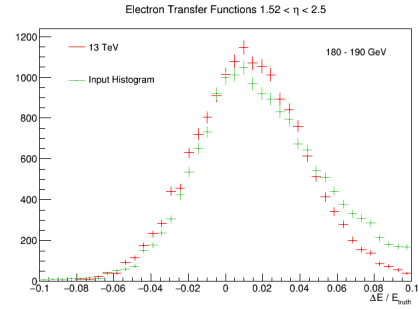
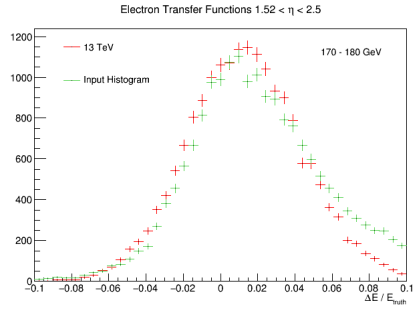
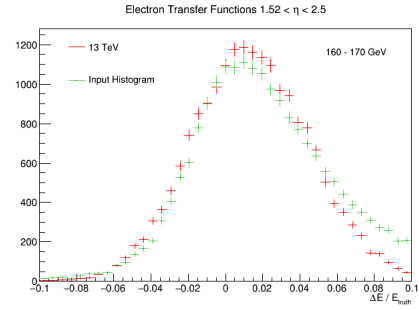
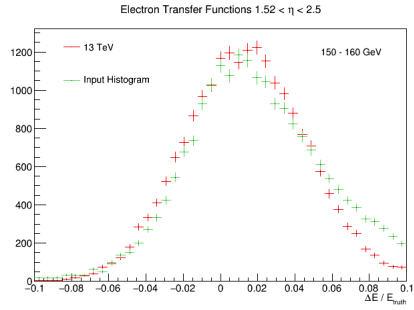
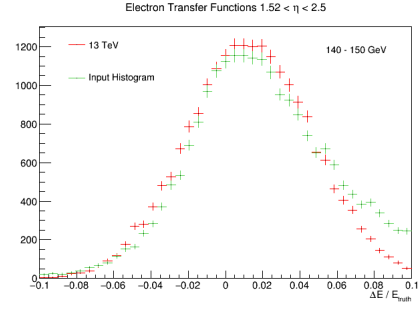
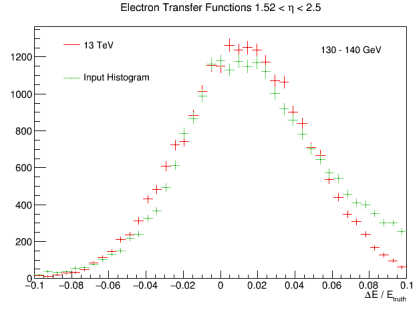


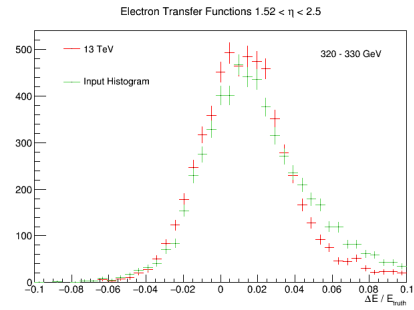
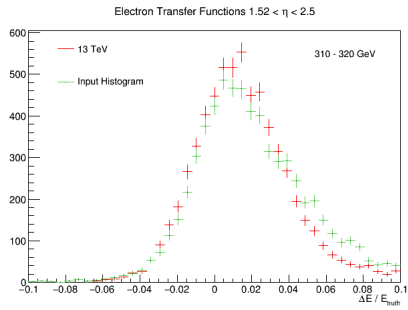
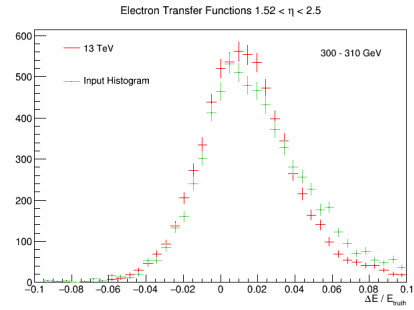
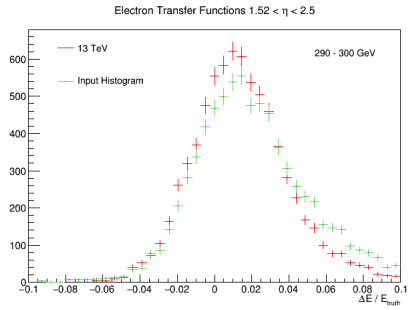
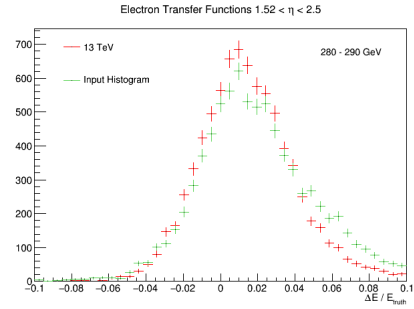
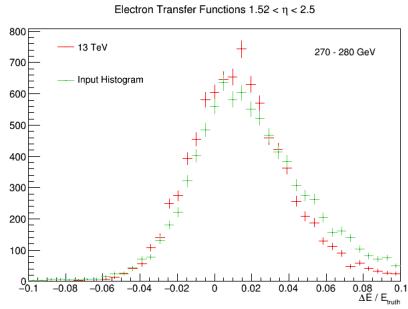
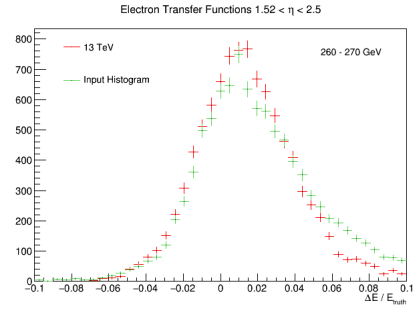
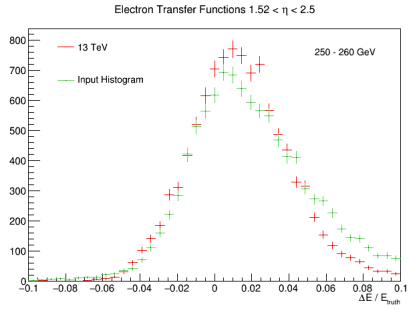
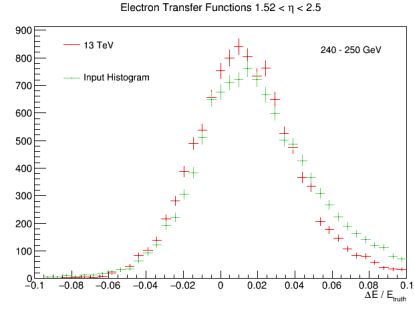
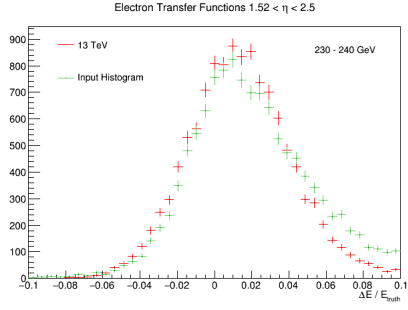


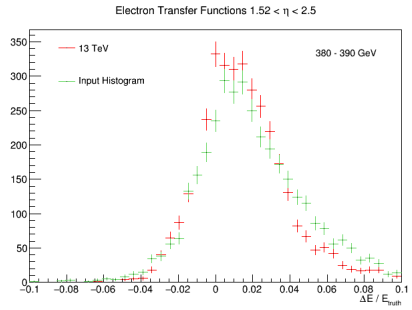
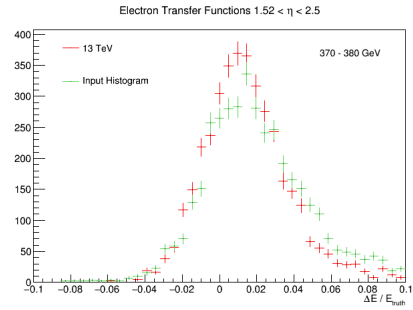
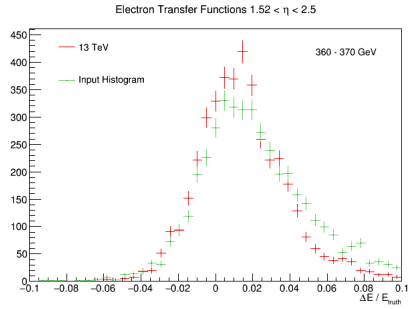
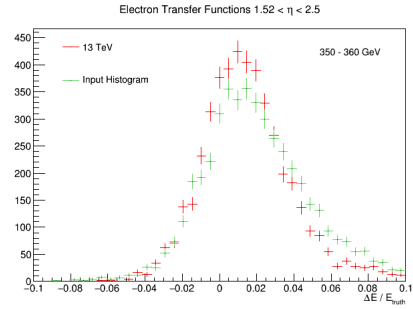
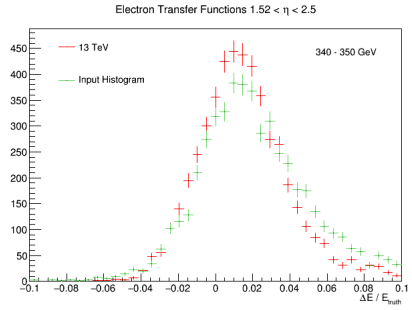
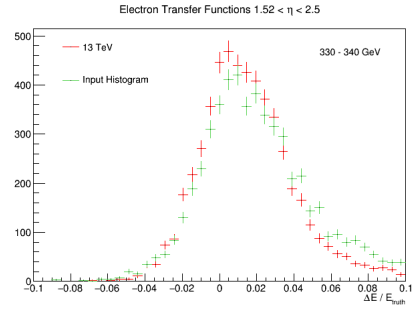
Eta Region 3

Transfer Function Parameters	
a_1	0.0107555
b_1	$-1.92758 \cdot 10^{-6}$
a_2	-0.00459803
b_2	0.448251
a_3	0.123156
b_3	0.000983448
a_4	0.0488765
b_4	-0.150587
a_5	0.0360965
b_5	$-1.54525 \cdot 10^{-7}$









6.4 Muon Transfer Functions

The functional form used for the muon transfer functions was

$$\frac{1}{\sqrt{2\pi}(\sigma_1 + p_3\sigma_2)} \left(e^{-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2} + p_3 e^{-\frac{1}{2}\left(\frac{x-\mu_2}{\sigma_2}\right)^2} \right)$$

where $x = \frac{p_{T,truth} - p_{T,reco}}{p_{T,truth}}$. Note that all energies are measured in units of GeV. The parametrizations used for the double Gaussian parameters are

$$\mu_1 = a_1 + b_1 p_{T,truth}$$

$$\sigma_1 = a_2 + b_2 p_{T,truth}$$

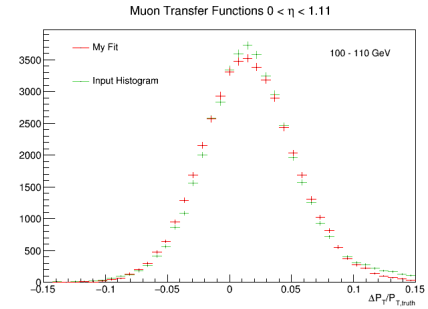
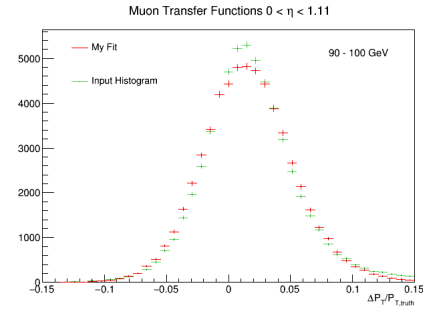
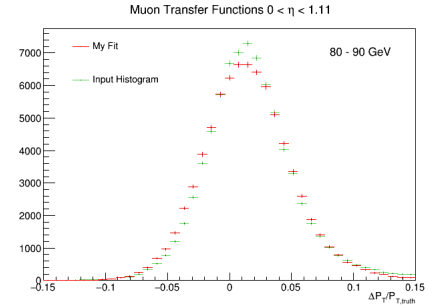
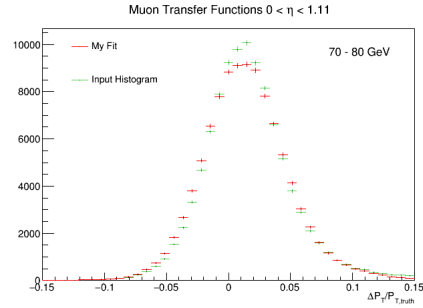
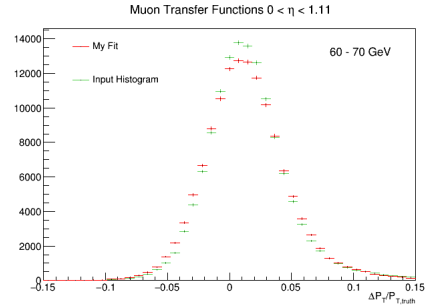
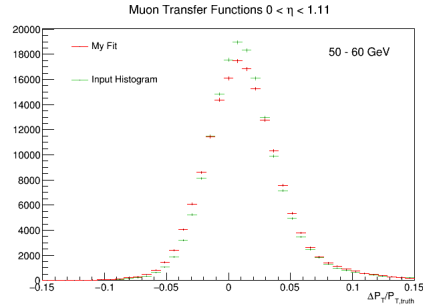
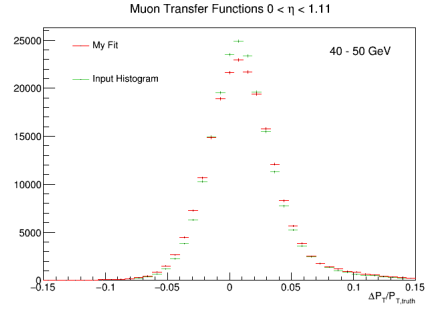
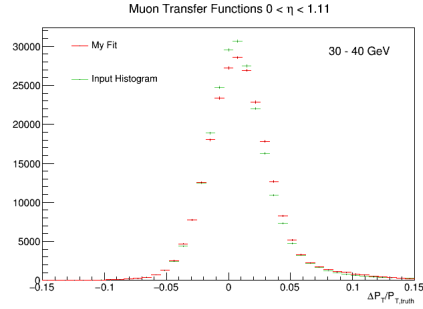
$$p_3 = a_3 + b_3 p_{T,truth}$$

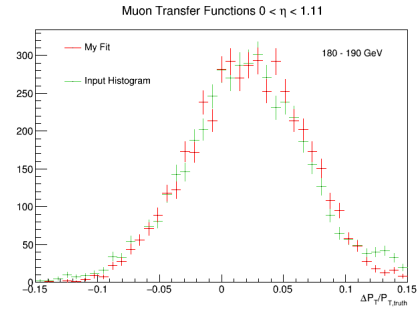
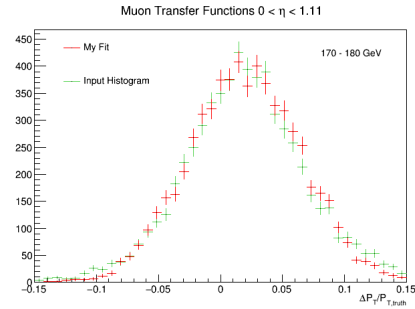
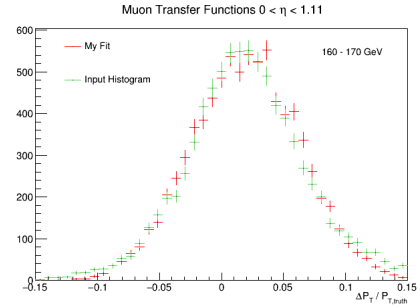
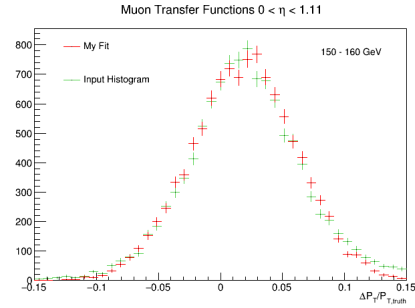
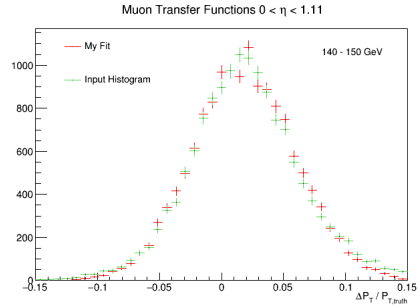
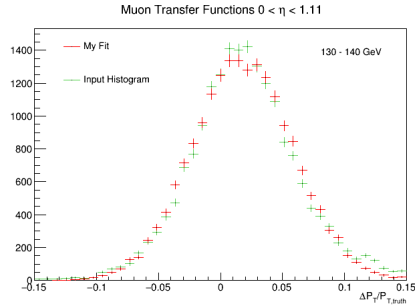
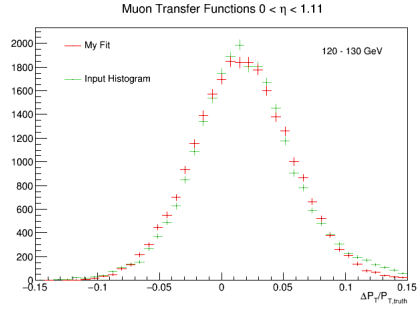
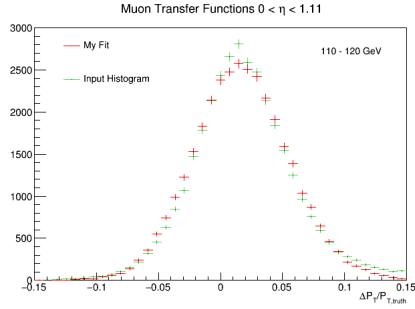
$$\mu_2 = a_4 + b_4 p_{T,truth}$$

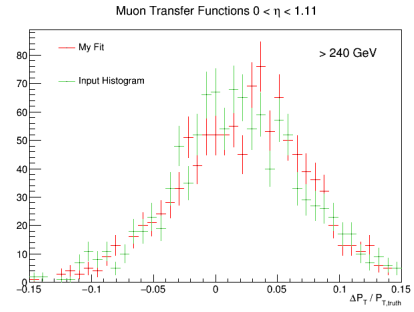
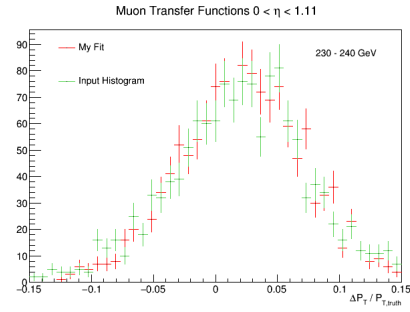
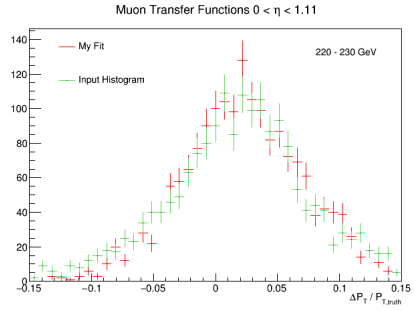
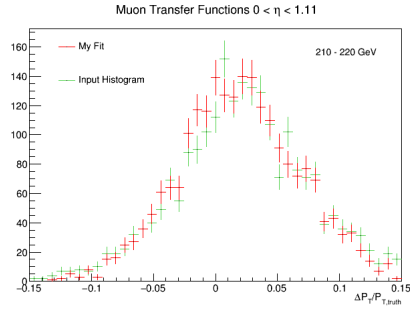
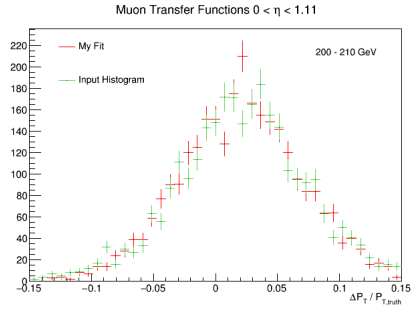
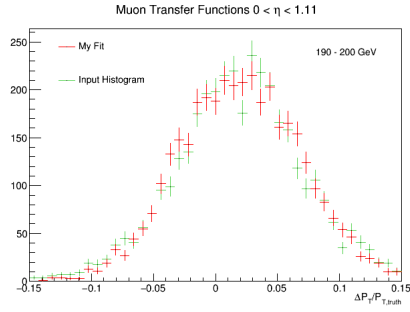
$$\sigma_2 = a_5 + b_5 p_{T,truth}$$

Eta Region 1

Transfer Function Parameters	
a_1	0.00385286
b_1	$8.31288 \cdot 10^{-5}$
a_2	0.0159891
b_2	0.000171618
a_3	-0.00333832
b_3	0.00195086
a_4	0.040049
b_4	$-9.60121 \cdot 10^{-5}$
a_5	0.056444
b_5	$-7.69121 \cdot 10^{-5}$

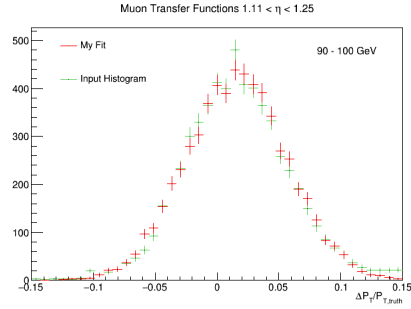
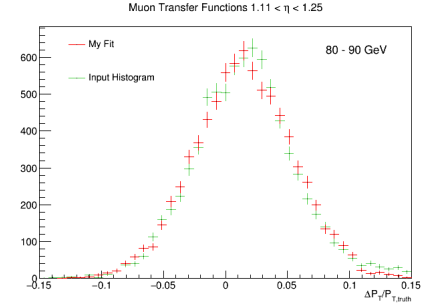
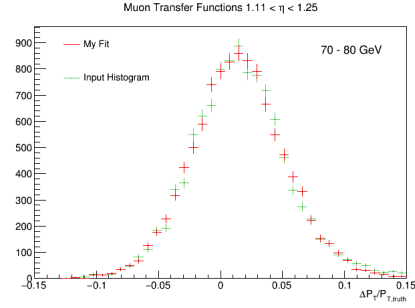
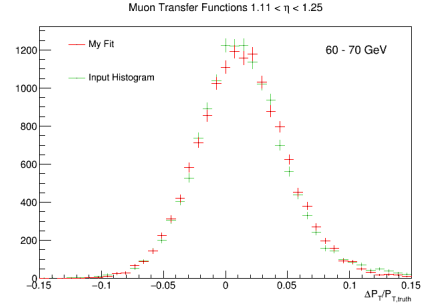
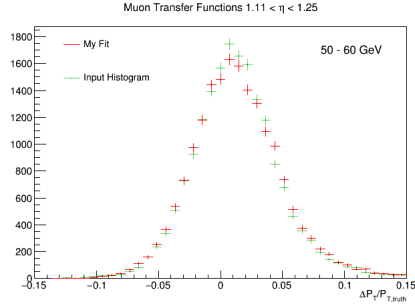
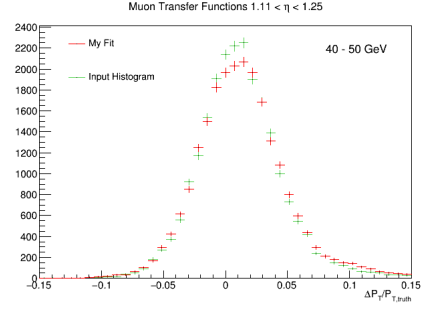
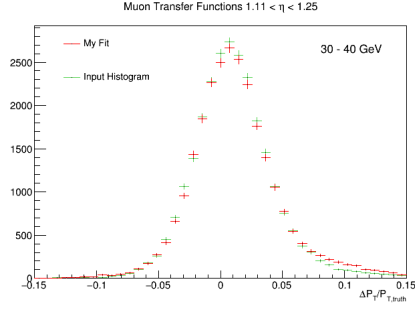


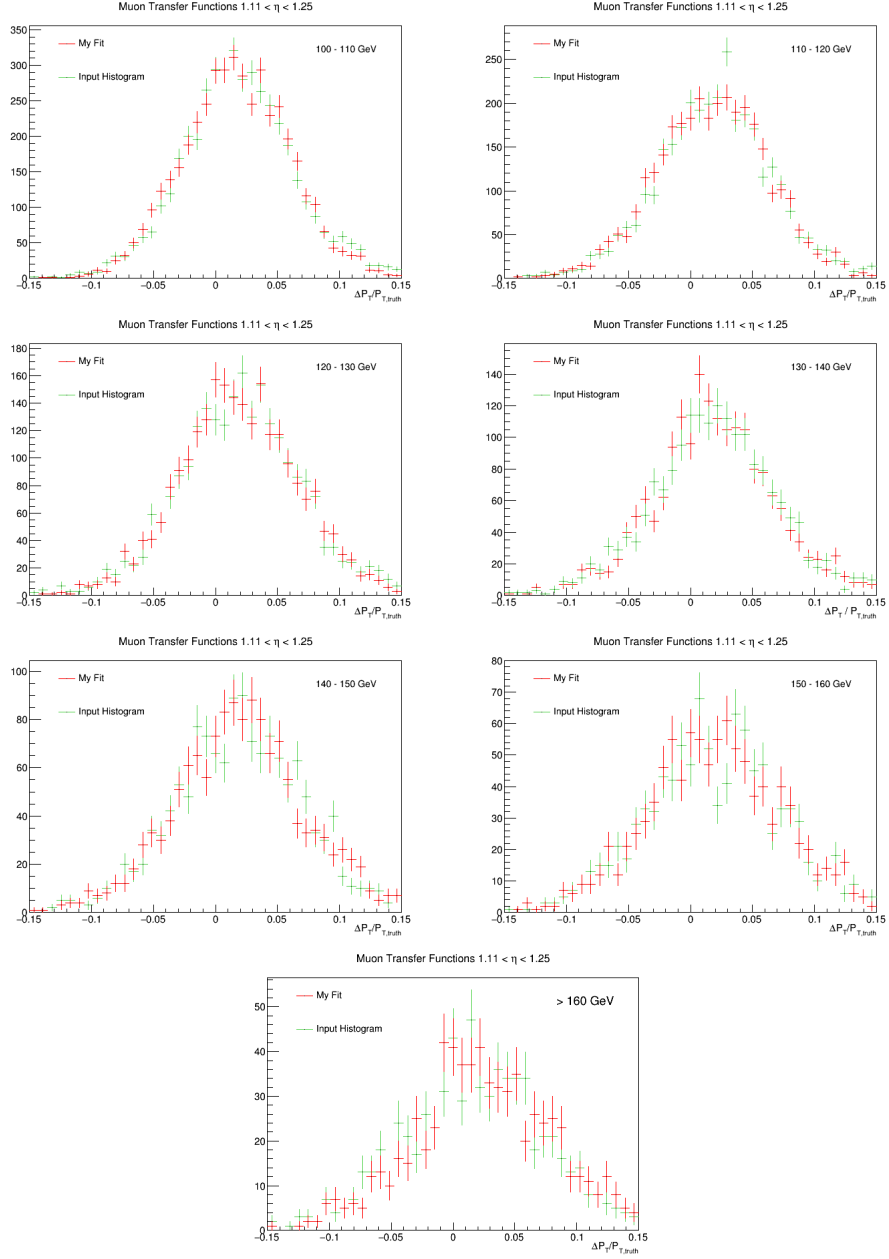




Eta Region 2

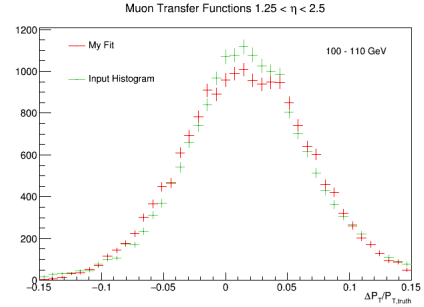
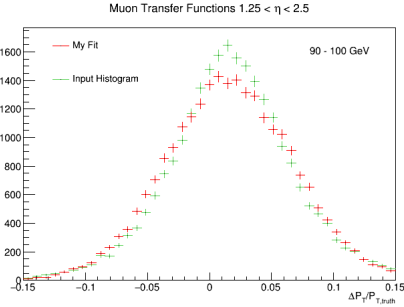
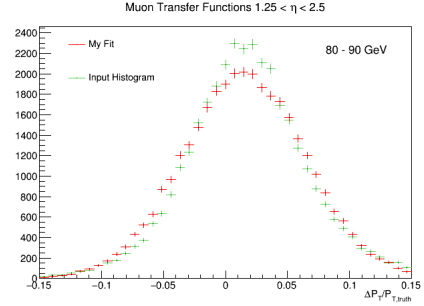
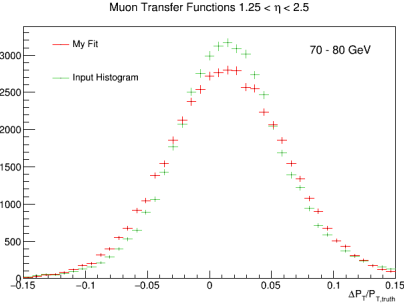
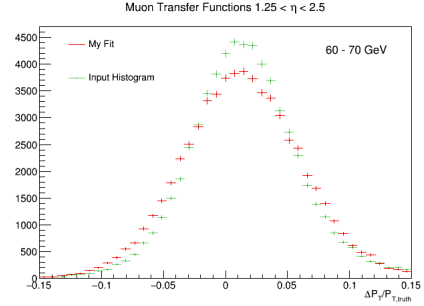
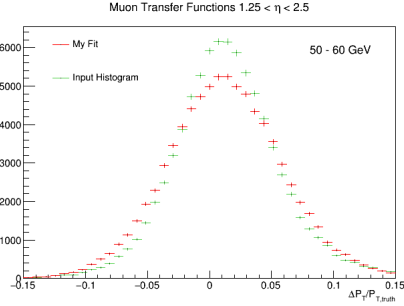
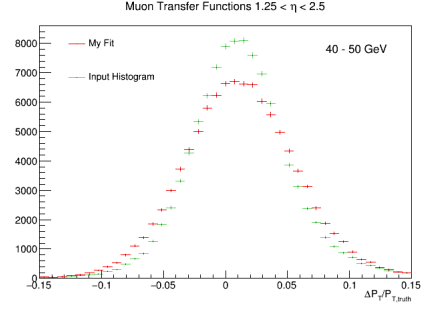
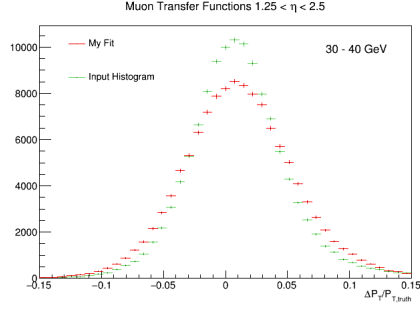
Transfer Function Parameters	
a_1	0.0022281
b_1	0.000126326
a_2	0.0150487
b_2	0.000275567
a_3	0.116811
b_3	0.000566139
a_4	0.037807
b_4	-0.000118265
a_5	0.0681063
b_5	-0.000284248

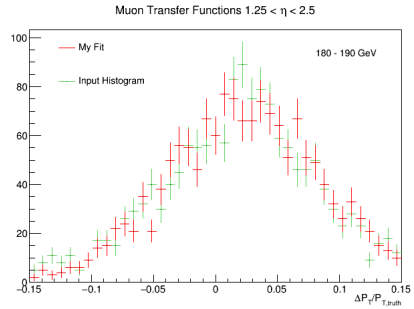
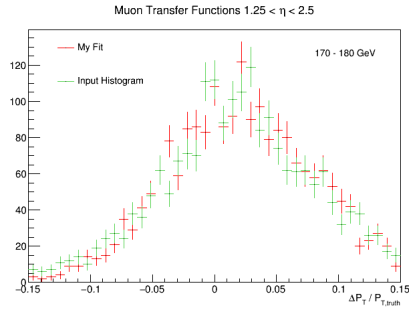
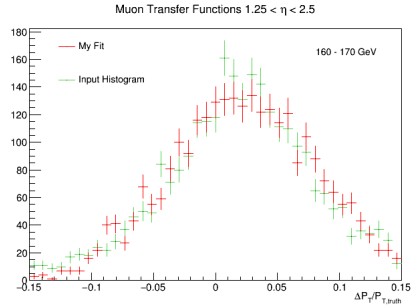
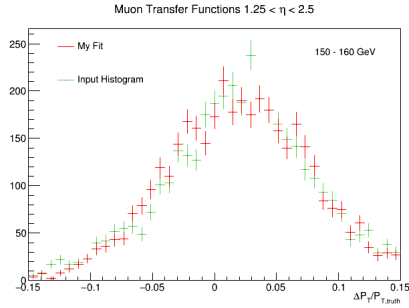
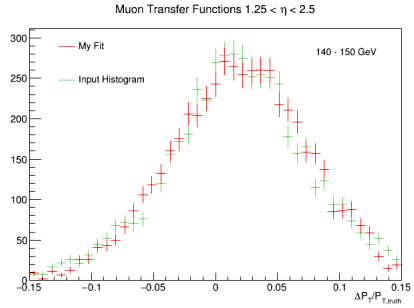
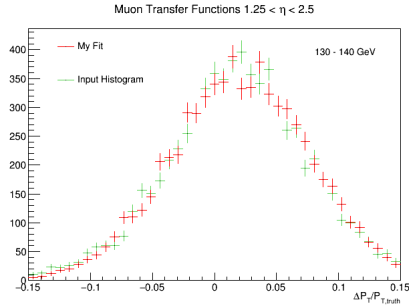
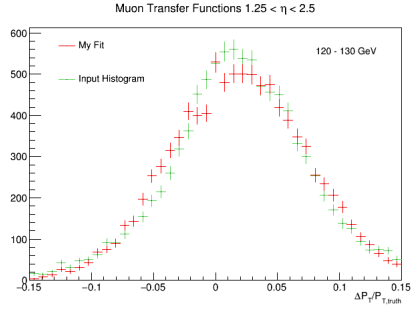
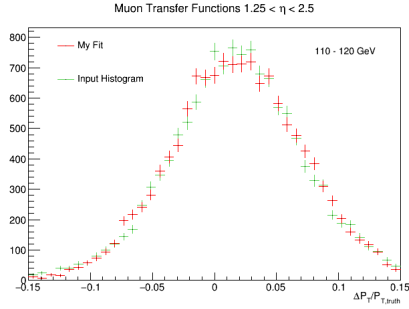


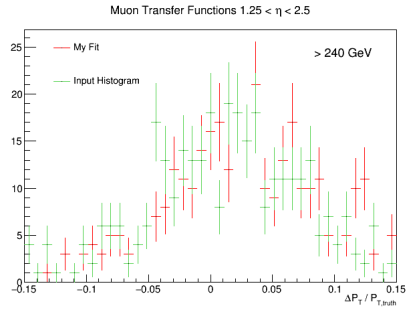
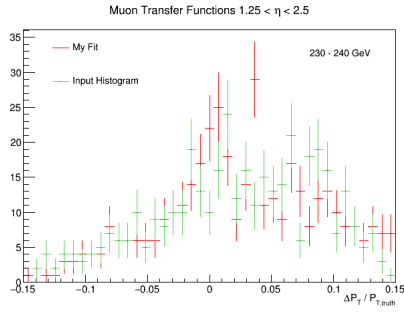
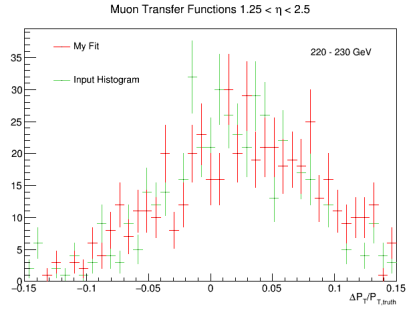
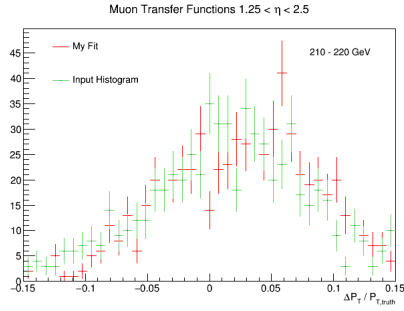
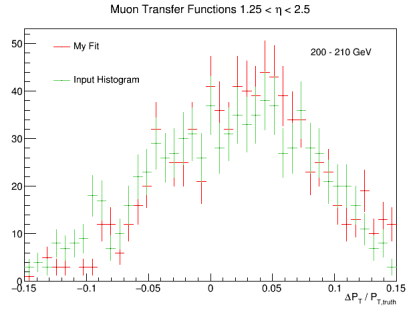
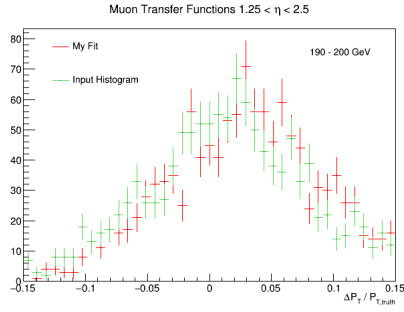


Eta Region 3

Transfer Function Parameters	
a_1	0.00287001
b_1	0.000125558
a_2	0.0307276
b_2	0.00018951
a_3	0.272824
b_3	0.000759595
a_4	0.0204579
b_4	$1.62599 \cdot 10^{-6}$
a_5	0.0597898
b_5	$-1.73917 \cdot 10^{-5}$







6.5 MET Transfer Function

The functional form used for the MET transfer function was

$$\frac{1}{\sqrt{2\pi}\sigma} e^{\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$$

where $x = \frac{E_{T,i} - MET_i}{E_{T,i}}$ where $E_{T,i}$ and MET_i are components of the neutrino transverse energy and missing transverse energy respectively. Note that all energies are measured in units of GeV. The parametrization used for the Gaussian width was

$$\sigma = p_1 + \frac{p_2}{1 + e^{-p_3(\sum E_T - p_4)}}$$

where $\sum E_T$ is the scalar sum of the transverse energy of every object in the event.

Transfer Function Parameters	
p_1	20.97
p_2	-4459
p_3	-0.1928
p_4	-3896

